OUR GOAL AT CMM:

TO ESTABLISH MEANINGFUL AND PRODUCTIVE RELATIONSHIPS BETWEEN ADVANCED MATHEMATICS AND ALL ENDEAVORS OF MODERN SOCIETY
The entrance to the School of Engineering at the Faculty of Physical and Mathematical Sciences, Universidad de Chile (1842), the host institution for the Center of Mathematical Modeling.
CMM

The Center for Mathematical Modeling was created in 2000 to encompass the explosion of ideas, projects and scientific activities resulting from our research. The CMM serves as a base for the intensive international collaboration, the increasing number of engineering and Ph.D. students in our programs and the enthusiasm and needs of our industrial partners. As a result the CMM enhances and enlarges the scope of our fundamental research and applications, opening new mathematical avenues for us and those generations to come.

The CMM was founded by members of the Department of Mathematical Engineering (DIM) at the Universidad de Chile with the aid of a fondap-conicyt project. From its inception, the CMM was an associated international unit of the CNRS, the first in mathematics outside of France. The center is physically housed in the Faculty of Physical and Mathematical Sciences which contains the oldest, most prestigious and productive engineering school in Chile. As a consequence our center has a strong scientific link with several departments in this school—industrial, mechanical, civil, electrical, computer science and earth science—and benefits from access to the 4,000 strong student population of the school and to more than 500 graduate students in diverse fields of science and engineering.
The mission of the CMM is to create new mathematics and to use it to solve problems arising from other sciences, the industry and government. This is done by a team that comprises a world-class group of leading researchers in mathematics. In fact, if we order mathematical institutions that have produced more than 300 papers in the last 10 years according to the impact their work has had, our university appears in 72nd place, preceded only by the best universities in the USA, Europe, one in China and two in Israel. This group is complemented by a highly trained team of scientists and a good number of the best Ph.D. and engineering students in mathematics that the country has to offer.

### The Top 100 Math Centers Ordered by the Number of Citations Per Research Paper
(last ten years)

Source: ISI Web of Knowledge, Essential Science Indicators

<table>
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<tr>
<th>Rank</th>
<th>Institution</th>
<th>Country</th>
<th>Papers</th>
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Number of papers produced by Math Centers with similar rankings to U. de Chile:
The first strategic objective of the center is scientific excellence and this guides its endeavors. The CMM is made up of 23 associated researchers from the DIM at the Universidad de Chile and 6 researchers from an associated unit in the Department of Mathematical Engineering at the Universidad de Concepción. Every year we receive more than one hundred visitors from all over the world, mainly from the usa, France and other European countries, Latin America, as well as from China and Japan.

We have 48 graduate students (30 at Universidad de Chile and 18 at Universidad de Concepción) from Chile and abroad pursuing their Ph.D.’s in applied mathematics. A number of the Ph.D. topics have arisen from very concrete industrial problems. We are also responsible for teaching more than 60 undergraduate students in Mathematical Engineering, arguably the most prestigious undergraduate career in our School. More than half of our graduates pursue Ph.D. degrees in the best universities in the world, and most of them return to the country to join departments of mathematics, economics and operations research in Chile and take up leadership roles in the scientific community. Our successful postdoctoral program complements this. The CMM is becoming an interesting place in the world for young scientists looking to start their careers in a competitive,
friendly environment that is well connected to applications. Every year we receive around 10 new postdocs from all over the world, mainly from Europe, Latin America, China and the USA.

Members of the CMM have a very active national and international network of collaboration. Indeed, more than 80% of the papers produced at the center are in collaboration with researchers from abroad: France (24%), the rest of Europe (24%), Latin America (23%), USA and Canada (11%) and our burgeoning cooperation with China (5%). More than one hundred researchers visited our center last year: France (46), the rest of Europe (22), USA (21), Latin America (21) and elsewhere (5). Besides our center is a member of the CNRS, France, which gives us prestige and unprecedented access to the French and European scientific communities. At the same time, our center serves as a platform from which many young Europeans, mainly French, have launched their scientific careers. Finally, the CMM belongs to an exclusive international network of centers of excellence in applied mathematics which includes: MITACS and CIRRELTS (Canada), PIMS (Canada-USA), MATHEON (Germany) and MASCOS (Australia). We also maintain especially strong links with U. Paris vi and E. Politechnique (France), IMPA and USP in Brazil.

Our center focuses on collaborative work between its members, applied mathematicians at other universities and researchers from

Over 100 researchers visited the CMM in 2006.

THE CMM’S PROMINENT INTERNATIONAL COLLABORATORS:

- CNRS FRANCE
- MITACS AND CIRRELTS CANADA
- PIMS CANADA-USA
- MATHEON GERMANY
- MASCOS AUSTRALIA
- PARIS VI FRANCE
- IMPA BRAZIL
- USP BRAZIL
other fields dealing with complex problems in engineering, biology and telecommunications, to mention just a few examples. Applied mathematics plays a vital role in science and technology. Engineering endeavors are already highly dependent on tools developed in applied mathematics and this dependence will only become more pronounced in the years to come.

The CMM also fosters contact between academia and industry, which leads to long-term research collaborations.

The CMM has a record of success in industrial problems. One of our major partners is codelco, the largest copper enterprise in the world, with its associated centers and startup companies, Biosigma and Micomo. We are working jointly with their researchers on several complex projects that are relevant to every facet of the company, ranging from bioleaching, rockbursting, fragmentation, converter processes and strategic/logistic planning. We also have worked in telecommunications with ministries in Chile, Colombia and Peru. In forestry, we have worked with Fundación Chile and the principle Chilean companies in the industry. The international flavor of the CMM has naturally encouraged us to develop projects worldwide.

Chuquicamata is the largest open pit copper mine in the world. Codelco, the company that runs it, is our main industrial partner.
As examples, we highlight our joint ventures with EDF, SNCF and Dassault, all of which are in France, as well as those with Alstom, Siemens, Bombardier, Hewlett-Packard and Microsoft. We also have important projects with the Chilean Ministry of Education and other state agencies. In Chile, our major industrial partners include LAN CHILE, Telefónica, Forestal Arauco, Metro, Enersis, Endesa and Antofagasta Minerals. In addition to meeting their specific challenges, we develop new and interesting mathematics based on these initiatives which results in papers, patents, algorithms and software.

Our community includes senior research scientists who are fundamental to applications. They are leaders in the development and execution of cutting edge and long-range research activities. As professional staff, the senior scientists nurture the intellectual life of the center by facilitating cross-disciplinary interactions and learning, by taking responsibility for project management, and by providing stability, continuity and a wealth of knowledge and experience. Just as the Faculty does, the senior research scientists collectively embody a breadth and depth of cross-disciplinary expertise.
The mathematical analysis and the computation of solutions for complex models in such areas as genomics, transportation and telecommunications, to mention just a few, demand new theories, algorithms and computational methods. The last issue is particularly important because real-world applications require us to deal with realistically sized networks having thousands of nodes, arcs and variables. Network structure holds serious implications for efficient computation and it is fortunate that such networks are often well suited for distributed computing by using multiple processors (ideally hundreds for our models). For this reason the CMM has created a laboratory of high performance computing. Two years ago, in a joint venture with Hewlett-Packard, we opened one of the most powerful clusters in the country (partially founded by Fundación Andes) that we expect to enlarge in the near future. We also participate actively in two international grid-computing networks, pragma and gelato, for which we form the Chilean node. Within the country, we give computational support to several universities and some public agencies such as the Chilean Bureau of Meteorology derive benefit from the cluster.

The creation of scientific knowledge and its dissemination is a primary responsibility of any research center. Furthermore, the members of the CMM believe this responsibility should extend to the national school system. Our country should kindle and capture the scientific imagination, curiosity and interest of our children and youth, lead them to higher standard of education and give them the skills and tools to understand a world
where science has an ever-increasing presence in our daily lives. The government and social leaders of Chile agree with us and have stated that improving mathematical education is a priority for the country. Our center assists in this effort by participating in a national project to revamp the university training received by high-school mathematics teachers. Accomplishing this task will involve the production of various materials for use by university professors in training teachers, including 15 monographs on a variety of topics in applied mathematics ranging from numerical analysis to statistics. In addition, we also advise the Ministry of Education on the setting of standards in the high-school curriculum and share and exchange experiences with the best educational centers in Europe and the usa.

As a recognized center of excellence in scientific research, the CMM is regularly approached by government agencies seeking a formal statement or expert opinion on scientific matters. At the same time, members of the CMM individually give advice to public institutions and policy makers. The kind of advice provided varies from responses to general scientific inquiries to aspects of educational policy. We also offer advice with regard to more specific matters such as energy management, where we have given technical support to the National Commission on Energy.

Some members of the CMM:
From left to right standing, Marcos Kiwi, Raúl Manasevich, Raúl Gouet, Jorge San Martín, Servet Martínez, and Alejandro Jofré.
From left to right seated, Juan Dávila, Michal Kowalczyk, Manuel del Pino, Salomé Martínez, and Jaime San Martín.
CNRS + CMM: A successful formula

The fruitful relationship with the Centre National de la Recherche Scientifique (CNRS), the most important scientific research institution in France and the largest in Europe, starts in the early 80’s long before the CMM was created. Since then, this collaboration grows becoming fluid and deep, generating many scientific works.

The joint work between French and Chilean mathematicians enriches the scientific collaboration between the two countries and settled the basis for an agreement between Universidad de Chile and CNRS. Under this agreement the CMM became the first international associated unit in mathematics outside of France and the fourth non-European unit after USA, Singapore and Japan, regardless of the discipline.

Consequences of this visionary association CNRS-CMM are: The joint Ph.D. training, the opening of emergent research areas and the co-organization of international congresses and schools. This partnership has been strengthened with the incorporation of Paris VI, one of the most productive and prestigious scientific centers in Europe.

This association allows the sojourn of 20 to 30 French mathematicians into CMM every year, where they develop joint research taking advantage from CMM’s special relation with local and international industries.

Key figures about the CNRS:

- **3.080 billion Euros** is its budget for 2007.
- **11,700** researchers work at the center.
- **85** exchange agreements with 60 countries.
- **1,340** are non-French employees in tenured positions.
- **268** International Programs for Scientific Cooperation (PICS).
- **15** International Joint Units (UMI).
- **9** CNRS offices are settled abroad (Beijing, Bonn, Brussels, Hanoi, Johannesburg, Moscow, Santiago de Chile, Tokyo, Washington).
- **7,450** patents belong to the CNRS portfolio. **1,057** are the active licenses.

Source: www.cnrs.fr

CNRS is the Centre National de la Recherche Scientifique, or the French National Center for Scientific Research. It is a government-funded research organization, under the administrative authoroty of the French Ministry of Research. CNRS is the largest of several French research organizations with the legal status of Public Scientific and Technological Institution. The CNRS was founded in 1939 by governmental decree. Currently it covers all the major fields of scientific research, and is organized into 6 research departments and two national institutes. The departments are: Mathematics, Physics, Earth Sciences and Astronomy (MPPU); Chemistry; Life Sciences; Humanities and Social Sciences; Environmental Sciences and Sustainable Development (I-oD); Information and Engineering Sciences and Technologies (I2ST). The two national institutes are: The National Institute of Nuclear and Particle Physics (IN2P3) and The National Institute of Earth Sciences and Astronomy (INSU).

Source: www.cnrs.fr
CMM is a very pleasant place to work, it has good facilities and a remarkable group of people. Somehow they look so relaxed and at the same time they are very productive not only in mathematics. On the other hand, the Center is outstandingly active and there is much international interchange.

Mike Boyle
Full Professor
University of Maryland, USA
What distinguishes this group—and which makes it more than the sum of its parts—is a common interest to study the mathematical and computational properties of discrete structures such as networks.

These structures appear in diverse areas of current scientific and technological interest. For example, real-world phenomena involving networks include urban traffic, the flow of electronic mail through the Internet, the spread of contagious diseases, the distribution of goods from warehouses to retail outlets and the self-regulation of cells.

In 2000, our group was young by scientific standards. Initially, we agreed on a medium-term strategy for developing scientific and teaching activities. So far, we have consolidated lines of individual research and established various international collaborative networks, first with groups in the USA and France, and later with Brazil and the Czech Republic. Our lines of research are centered on topics such as graph theory, the foundations of computation, algorithms and complex systems. Beyond just using common language and techniques, a common interest has crystallized within the group, namely the study of networks. This common interest is equally concentrated on combinatorial and structural topics and aspects concerning applications. The enormous relevance of networks as a subject, from both a theoretical and practical viewpoint, explains in part the extensive national and international collaboration that the group has maintained since its creation. For example, in the school of engineering at the Universidad de Chile, our members co-supervise the engineering and doctoral theses of students studying transportation systems, computer science, operations research and biotechnology. The group receives financing from a diverse range of sources such as FONDECYT projects, ECOS, MILLENNIUM NUCLEI and, of course, FONDAP. In 2004, we jointly organized the Latin American Conference on Combinatorics, Graphs and Applications with the Department of Industrial Engineering at the University of Chile.
Recently, many of our students who traveled abroad to undertake doctoral studies have returned to the country while other young scientists we trained have joined academic groups similar to ours at various institutions. In addition, we have strongly expanded our activities since successfully obtaining our first group project: Networks in Mathematics and Sciences of Engineering (Project Anillo).

Through the project, our group became associated with the combinatorial optimization team at Universidad Adolfo Ibáñez and, in doing so, became a regional leader in the development, application and dissemination of discrete mathematics and theory of computation. As an example, we organized the Latin American Theoretical Informatics Symposium in 2006. To measure the impact of this event, it is worth emphasizing that 224 original works from over 20 countries were received, an unprecedented figure in the region and of which only 25% were accepted. There are many other facts reflecting the standing of the group, for example: the arrival of two foreign postdocs, the one year sabbatical of a researcher from the prestigious discrete mathematics group at Charles University (Prague), the incorporation to our group of a young scientist from the CNRS (France) for a period of two years, the co-supervision of doctoral students and the departure of students to pursue doctoral studies overseas at institutions such as MIT and University of Paris vi. Another relevant achievement has been to merge two important conferences—the Brazilian GRACO and the Chilean LACGA—and to organize the most relevant Latin American event in the area, Latin American Algorithms, Graphs and Optimization Symposium (Puerto Varas, November, 2007), in conjunction with the Department of Industrial Engineering at the Universidad de Chile.

The relevance that discrete mathematics has attained in computer science and the expanding spectrum of applications in engineering are also a great opportunity for the group to grow and develop in order to succeed in meeting the needs of academia as well as the national industrial sector.
Our center hosts a significant proportion of the critical mass of researchers in mathematics, engineering and the applied sciences who have expertise in the mathematical modeling of systems governed by partial differential equations. The Mathematical Mechanics research team includes researchers in mathematics and engineering, Ph.D. students, final-year undergraduate students, postdocs, engineers and a global network for scientific collaboration which includes researchers who are leaders in various areas. In our opinion this group stands on two fundamental pillars:

– The work done by our researchers in distinct areas of mechanics and mathematical physics, their applications and deployment in industry. This work encompasses such disciplines as fluid mechanics and solid-liquid interaction, materials science, metallurgy and areas specific to mathematics such as homogenization, optimal design, control (both exact and approximate) and inverse problems;

– The knowledge that has been accumulated in the country about numerical methods, algorithm design and advanced computational tools for simulating and calculating solutions to applied and industrial models.

**Research**

The group has given rise to outstanding contributions to areas in applied mathematics and modeling which has facilitated strong interactions with diverse areas in mechanics and mathematical physics. The breadth, depth and quality of this achievement places the group in a prime position in the international scientific community, acknowledging our seminal work in numerical and mathematical analysis in fluid mechanics and fluid-structure type interactions, besides having established the foundations of and developed the theory on which Bloch analysis in homogenization is based.

The impact of our research is reflected by the number of references to our publications made by influential mathematicians in their books, as well as citations in publications derived from our works which appear in the specialist literature. Our works registered more than 450 citations of articles and books. As a further indication of the importance of our work to the international mathematical community, members of the group have been invited to deliver plenary talks and inaugural presentations at renowned international mathematics meetings.

Over the years, we have established an international network of collaboration that includes researchers from different institutions and universities in France, Spain, India and usa. In Latin America we highlight our collaboration with IMPA and University of Campinas.

**Education**

Possibly one of the strongest points of the group is the education and training available to students and young researchers in mathematics. In fact, we have succeeded in creating a school for mathematical mechanics which is internationally recognized. At the national level, the school has proponents in numerous universities throughout the country (the Universidad de Chile, Pontificia Universidad Católica de Chile, Universidad de La Frontera, Universidad de Concepción and Universidad del Bío-Bío) while internationally, supporters are to be found in France (Université de Caen), Spain (Universidad Autónoma de Madrid), Rumania (Universities of Bucarest and Pitești), India (Indian Institute of Technology) and Mexico (Instituto Tecnológico de Querétaro).
We have played a founding role in the Universidad de Chile’s Ph.D. Programs in mathematical modeling and fluid dynamics. This participation has proven to be most fruitful, with a total of 8 Ph.D. defenses having taken place during the past 5 years, which represents a significant number of theses defended during this period. The situation is similar for students doing their thesis as part of the Mathematical Engineering Degree; under our supervision, 9 students have graduated and we currently have several students.

Graduates interested in starting research careers find the postdoctoral program in the CMM very attractive. Indeed, our group has supervised several young researchers from Chile and abroad: from Universidad de Chile (2), University of Pitesti, Université Paris-Dauphine, University of Madras, Université de Caen, Université Jean Monnet, University of Bucharest, University of Nancy 1, École Nationale des Mines de Saint Etienne, Indian Institute of Technology and Universidad del Bío-Bío. All of them now have a research position either in Chile or in their original home country.

The head of the group has been the principal researcher of the MECESUP project Optimization of the academic management and design for the development of general education integrated with undergraduate studies at the Universidad de Chile since its inception in 2001. This project, with a budget of USD2,000,000 per year and scheduled to last 8 years, has played a central role in undergraduate reforms implemented in the Universidad de Chile over the last decade.

**Links to Industry and Other Organizations**

The group of researchers in mathematical mechanics has a long history of interaction as much with industry in Chile as with industry overseas. An initiative which illustrates this is our FONDEF project on the fluid-dynamics of fusion, conversion and refining of copper which was carried out over three and a half years in cooperation with CODELCO and IM2. Contributions by the IM2 engineers were critical in suggesting modifications to the design of the Teniente-converter.

More recently, members of the group have been awarded the following applied contracts which are currently in progress: The Mathematical Modeling of In Situ Leaching Technology (with CODELCO) and the Numerical Simulation of Ventilation Conditions in Trains with METRO. Also we have opened the Mining and Metallurgy Modeling Laboratory (LMM3) to consolidate a multidisciplinary technology development team which focuses on providing decision-making support in important investment plans elaborated by CODELCO.
Our research efforts are primarily focused on the qualitative study of solutions to nonlinear partial differential equations with emphasis in elliptic or parabolic. In general, we work towards understanding the structure and asymptotic properties of sets of solutions to specific equations. Problems of the type that interest us arise in diverse fields such as material science, astronomy, the theory of combustion, superconductivity and mathematical biology. On the other hand, an understanding of these problems has led to the development of powerful tools in different branches of mathematics, for instance, such knowledge was a key ingredient in the recent resolution of the famous Poincaré conjecture. Such problems have posed questions which, while being easy to state, are difficult to solve and have historically driven developments in mathematical analysis, giving rise in recent decades to advances in the calculus of variations and nonlinear functional analysis, subjects at the interface between analysis, geometry and topology. In recent years, scientific computation has proved to be a powerful ally in the study of these problems, and with numerous Chilean and international collaborators, have attacked problems along several lines and publishing more than 130 articles during the last 6 years. A significant number of these appear in the most important journals in the area and general mathematics. Our work is well-recognized by specialists worldwide, its impact being quantified by approximately 1,500 citations in the ISI database, the majority of which have accrued during the last few years.

We have organized 3 large conferences: A joint United States-Chile conference on nonlinear analysis and two IMA events on partial differential equations. In addition, we have been involved in organizing Pittsburgh-Sobolev-Gagliardo-Nirenberg and Gross-Sobolev optimal embeddings in the asymptotic behavior of equations for rapid diffusion in a porous medium. In addition, we have obtained new results in blow-up phenomena and vanishing in finite time. In another line of work, we have studied models of Brownian ratchets and their transport phenomena pertinent to molecular biology. We have studied competition systems relevant to ecological modeling, namely, segregation and stability induced in spatially inhomogeneous crossed diffusions. Furthermore, we have obtained fundamental results for evolution equations in which the diffusive term is replaced by non-local...
dispersion operators, objects which are natural to use in models but which are not well understood mathematically.

**Singular Perturbation**
This deals with problems having parameters which induce the formation of solutions with patterns of concentration in the form of measures or singular solutions supported on sets of lower dimension when taken to their limits. In several works, we have examined the semi-classical limit in nonlinear Schrödinger equations. Also, a conjecture about the presence of solutions concentrated on curves was validated using methodology that was also employed in the construction of multiple interfaces for the Allen-Cahn equation.

**Criticality**
The role of critical exponents for Sobolev embeddings in the solvability of elliptic problems which involve the Laplacian operator has long been known to mathematicians. Much less clear is the meaning of this exponent in nonlinear problems, as well as linear problems when the form of divergence is not present. We have considered extremal Pucci operators and defined a new notion of criticality whose implications have been explored in various articles, at the same time introducing a new line of investigation dealing with the solvability of these problems by means of viscosity solutions.

**Problems with p-Laplace Operators and their Extensions**
This line of research focuses on equations and systems involving second-order operators having nonlinear forms of divergence, specifically the $p$-Laplacian and non-homogeneous extensions ($\phi$-Laplacian). These studies are motivated by problems arising in mechanics in continuous media. We recently showed that the first eigenvalue in the vector case coincides with that in the corresponding scalar problem.

**Training**
We have directed an important number of students at master and Ph.D. levels in recent years, with theses giving rise to publications in good journals. Most of master students have continued into Ph.D. studies in our field, in some of the most prestigious centers in the world. Several of our former students are currently active researchers in Chilean universities.

**Education**
Members of our team leads the CMM initiative on education at the high-school level. We are producing a series of monographs which will be used in the high-school teachers’ curriculum. These include classical topics in mathematics as well as more applied areas like statistics and optimization. We have also been instrumental in producing standards for the high-school curriculum and the material that came out of this project is being used by the Ministry of Education.
In recent years, our group has made several important advances in Numerical Analysis. This was done in conjunction with other researchers in Chile and abroad. More precisely, we have developed new theoretical results and corresponding computational tools which have allowed us to derive efficient solution procedures for a wide class of linear and nonlinear problems arising in potential theory, electromagnetism, elasticity, fluid mechanics, fluid-structure interactions, sedimentation processes, wave propagation and other areas. Our results, which involve well known procedures such as finite elements, boundary elements, finite volumes, and stabilized and discontinuous Galerkin methods, have been published in a wide range of journals for mathematics and other disciplines with a high impact factor.

In addition, new research areas have emerged from the work carried out by this group, of which we mention just a few: Augmented and stabilized finite element methods for elliptic and parabolic equations; mixed finite element methods for non-coercive problems; multiresolution schemes for degenerate parabolic equations and PML methods for scattering problems.

Our focus is on the mathematical modeling of various physical problems and their computational challenges. Subjects that we have studied include: Mathematical modeling and numerical simulation of vibrations in coupled systems consisting of elastic structures interacting with compressible fluids; mathematical modeling and numerical simulation of damped transmission of vibrations between different media separated by thin layers of a viscoelastic fabric; analysis and numerical solution of active noise control problems; mathematical and numerical analysis of mixed variational formulations for linear and nonlinear transmission problems on interior and exterior domains; analysis of efficient finite element methods to solve Maxwell equations in the low frequency regime; development and analysis of PML absorbing boundary conditions; a posteriori error analysis to control the discretization errors and adaptive algorithms to automatically generate efficient Ansatz spaces; efficient solution procedures for the arising discrete systems, including algorithms and analysis of preconditioners; high order boundary element methods to efficiently approximate the unknowns associated with the boundary integral operators; analysis, numeric and control of quasi-geostrophic ocean models; numerical analysis of nonlinear hyperbolic equations with relaxation terms and application to chromatography and distillation models; parameter identification of degenerate parabolic equations modeling sedimentation problems; mathematical modeling of traffic flow using conservation laws and hyperbolic equations; modeling of pollutant dispersion and degradation in aquatic media.

Our group is one of the most numerous and productive in Southamerica in the field of numerical analysis, measured by the number of ISI papers, and our work has a significant impact measured in terms of citations. As a consequence of this international recognition, members of our group serve on the Editorial Boards of important ISI journals like *Numerical Functional Analysis and Optimization* and *Applied Numerical Mathematics*. Apart from this we organize international events for researchers and advanced undergraduate and graduate students enhancing our international visibility. Following the success of WONAPDE 2004 and WONAPDE 2007, each boasting over 100 participants from various countries.
and a special issue of the ISI Journal *Applied Numerical Mathematics* devoted to its proceedings, we plan to continue holding this workshop every three years so that it may become a traditional event among the international community of Numerical Analysis.

We maintain very close collaboration with researchers from various institutions abroad, including the Universidad de Oviedo, University of Hanover, University of Minnesota, Universidad de Zaragoza, Universidad de Santiago de Compostela, INRIA-Rocquencourt, Laboratório Nacional de Computação Científica (Lnccc), University of Oslo, Otto-von-Guericke-Universität Magdeburg, University of Stuttgart, Mount Allison University, MiraCosta College, Brunel University, Universidade Federal de Santa Maria and IMPA.

**Education**

Our primary contribution to the training of new young scientists is through the supervision of thesis works that form an integral component of the professional degree in mathematical engineering and the Ph.D. Program in applied science with major in mathematical engineering hosted by the Department of Mathematical Engineering at the Universidad de Concepción. The Ph.D. in mathematical engineering we offer is an interdisciplinary program and some of our Ph.D. students are Chemical, Metallurgical, Civil or Electrical engineers. Apart from this, researchers from this group also advise undergraduate and graduate students in other courses and programs from our university and elsewhere.

There is a rising demand in Chile for young researchers specialized in Numerical Analysis and Scientific Computing. All ten students who have graduated from the Ph.D. Program and who were supervised by members of this research team have been hired by Chilean institutions right after obtaining their degrees. Some were even offered positions before they had completed their studies. We are gratified to see our training efforts so well rewarded.

**Industrial Links**

Researchers from this group took part in a FONDEF project, jointly undertaken with Centro EULA-Chile, the Department of Chemical Engineering at our university, and several industries in the region, on river water quality model for the assessment of waste water and flow modification effects. In addition, some of our researchers have been involved in industrial projects with foreign organizations. For instance, we contributed to a joint project between Universidad de Santiago de Compostela and the company Ferroatlántica I+D concerning the mathematical modeling of an arc furnace for silicon production. Closer to home, a couple of our members collaborate with the Department of Metallurgical Engineering along with a number of students from various Ph.D. Programs at the Universidad de Concepción on mathematical problems related to the processing and extraction of metal from copper ores. In particular, they work on mathematical models for heap leaching in copper ores, inverse (parameter identification) problems for sedimentation models and models for the design, simulation and control of industrial thickeners. The expertise developed during this collaboration has been invaluable in several projects with the mining industry. In addition, the parameter identification problem is conducted in coordination with LUM GmbH, a Berlin-based laboratory centrifuge manufacturer.
Everyday, individuals, enterprises and governments need to make decisions in an increasingly sophisticated world. Like never before, technology offers a great variety of alternatives, from the latest generation of cellular telephones to the most powerful supercomputers, all communicating across complex networks, providing access to more and better information for making decisions, and supplying a broader spectrum of feasible possibilities to implement.

So as not to get lost in this complex scenario while striving to minimize costs, travel time, risk and/or error, or when one wishes to maximize benefits, flows, efficiency or reliability, one requires a methodology based on a solid foundation.

Optimization is the science of how to make the best decisions. A decision is made by comparing different alternatives from among a set of feasible options according to some criteria that it is desirable to optimize. When we are confronted by diverse criteria in competition, it is necessary to determine an equilibrium solution. Part of the work carried out by members of the Optimization and Equilibrium Group involves the mathematical modeling of systems requiring optimization. These activities can range from planning the operation of a mine to the design of incentive mechanisms in the electricity generation market, through the sharing of limited resources such as a public transport system between users or a communication network by clients and companies in the telecommunications sector, to give only a few examples.

Unfortunately, there is no universal method that allows for the efficient solution of all the optimization problems that can be posed. Nevertheless, we avail ourselves of highly capable algorithms to attack a great diversity of problems, albeit resorting to approximation in some situations. Before deciding which algorithm to use, it is first necessary to have an understanding of the specific problem we wish to solve, such as the objective measure to optimize, the nature of the restrictions the solution must satisfy, what is known about the quality of the available data and how sensitive the problem is to perturbations in its parameters.

Problems frequently exhibit a mixed nature composed of diverse characteristics. This together with ever larger problems requires a complete knowledge of existing techniques and the ability to develop new algorithms and analyze and interpret their results. These are skills that we cultivate in ourselves and students under our supervision.

The interdisciplinary character of optimization in the CMM can be seen in the work that we do. For example, it is often necessary to manage situations involving elements of uncertainty, a phenomenon that is readily modeled by means of stochastic processes. Also, there are many situations where the system being optimized evolves through combinations of states which
can be handled using discrete mathematics. Not only do probabilistic and combinatorial techniques appear in modeling, but they are fundamental tools for the design of efficient algorithms.

Continual interaction with members of other groups, both inside and outside of the CMM, is essential for members of the Optimization and Equilibrium group. This both enriches the potential of our accomplishments and enhances our ability to supply better and more flexible answers to the increasingly difficult challenges that we face.

**Industrial Problems and other Initiatives**

Nowadays, Optimization and game theory are evolving together in several aspects, not only as fundamental areas but also in many applied problems. Congestion and public transportation, energy and power market, telecommunication and regulation, fishing and population dynamic, forestry and planning, revenue management and pricing, rock mechanics and short-term planning are all good examples of this joint evolution. The optimization group has been involved in all these applications adding always an industrial or public institution as a partner such as Codelco, the National Commission of Energy, Telecommunication regulators, Fishing Institute (IFOP), energy regulation offices in different countries in South America and the airline company LAN Chile. The impacts that these collaborations have produced are wide and deep, among them a significant reduction of costs and improvement on the productivity for the cooper mining and energy industry, improvements on economic regulation models for the telecommunication and energy industry, which produced an important impact on the Chilean society and other countries in the region such as Peru and Colombia. More recently part of group have been interacting with the company Hewlett-Packard on the area of "utility or on demand computing" producing new models for generating optimal contracts in the framework of this future attractive business. Furthermore, the optimization group has been leading the high performance computing initiative promoting the interaction with international Consortium and Federations as PRAGMA and GELATO, in which are involved most of the main informatics companies as Intel, Hewlett-Packard and SUN. Finally, one member of this group has been leading the initiative "Institute for research on education" at Universidad de Chile.
Probability theory is the branch of mathematics that deals with uncertainty. Since the 17th century, mathematicians have created a body of concepts that formalized our intuition on chance and a way of handling it. Two of its greatest achievements are the law of large numbers and the central limit theorem which are at the heart of science. In the last few decades probability theory has started to play a central role in other scientific areas. Large deviations and concentration of measures constitute an important corpus of concepts and tools in statistical mechanics, stochastic calculus is at the basis of mathematical finance, cut-off and perfect simulation are central in discrete random structures and ergodic theory and symbolic dynamics play an important role in computer science and bioinformatics. All of these fall under our areas of interest.

Our group has scientific collaborations with scientists from well known international centers. In addition to individual scientific relationships, we have rich and deep interaction with the following centers: Paris VI, Institut de Mathématiques de Luminy, École Polytechnique, Instituto de Matematica e Estatistica, Universidad de Sao Paulo, Cornell University, University of Maryland, IMPA and University of Bielefeld. Together with these centers we actively develop joint research, foster interchange of students and organize international meetings. As individuals we have good and productive relations with specialists from the USA, Europe and Latin America.

Our theoretical and industrial research receives an important support from Millennium Nucleus Information and Randomness.

Killed Processes
A killed process is a process stopped at the boundary and it is an effective mathematical tool to study the population size of biological species. Related notions are the survival rate and quasi-stationary distributions. Our group has made substantial contributions in this area. Our main challenge now consists of extending these results to some basic models arising in mathematical ecology.

On the other hand, the study of the asymptotic behavior for the heat kernel on multidimensional unbounded domains with Dirichlet boundary conditions is a very hard and classical problem in analysis and PDE. We have solved this problem for several cases, attracting the attention of specialists in potential theory and PDE’s.

Processes with Summable Decay Memory
In many applications, processes with long range dependence and processes with infinite memory are being studied more frequently because of their relevance. In this direction, our contributions in the summable memory decay case are the construction of regeneration times, and the standardness property of the associated filtration.

Discrete Random Structures
In computer science, transport or telecommunications, there is always a discrete random structure at the core of the problem. The main challenges here are the study of urn models, probabilistic analysis of algorithms, the packing and parking problems, scenery reconstruction and bioinformatics. The main theoretical tools we use to understand these problems are martingale theory, large deviations, analysis of generating functions and combinatorial techniques.
Dynamics of Low Complexity Systems
The main topic is the study of continuous and measurable eigenvalues of Cantor minimal systems. The class of linearly recurrent systems is the main objective of our work. We have provided necessary and sufficient conditions for a complex number to be an eigenvalue of a system in such a class, both in the continuous and measurable case.

Backward Stochastic Differential Equations
The main application for these equations is a numerical scheme that is used to approximate a Black-Schole’s model when an explicit solution is not available.

Industrial Projects
We have two main industrial endeavors, mining and forestry. In mining we face two challenges, bioleaching and fragmentation. Concerning the former, we have created a Laboratory of Bioinformatics and Mathematics of the Genome (LBMG). Our laboratory is the theoretical counterpart to wet laboratories at BioSigma s.a., a start-up of CODELCO and Nippon Mining. Several patent applications by the LBMG are currently in process. With IM2 at CODELCO, we have investigated a way of measuring the efficiency of blasting and crushing in the mining process. By using image analysis we handle several types of biases to estimate the granulometry size distribution.

In forestry we have studied the effects on pine plantations of a plague caused by the pine shoot moth. Jointly with Fundación Chile, and the main Chilean forestry enterprises, we have developed models that these companies now use in their tactical and strategic planning. In both of these endeavors, we have licensed patents on the main process, in addition to producing research papers.

Stochastic Simulation Laboratory
Created in 2006, this laboratory was designed to answer an increasing demand from the Chilean industry and engineering sciences for numerical solutions of problems where random phenomena play a fundamental role.

Recently the laboratory has obtained a 2 year contract with Micorno to develop data analysis and simulation tools to measure and predict strain propagation in block-caving mines using optical fiber technology created by NTT.

Training
We attract and support a very active group of undergraduate and graduate students involved in theoretical and applied problems. At the undergraduate level we introduce them to the modern theories of stochastic processes, dynamical systems and information theory, including applications like finance, bioinformatics, statistics and networking. At the graduate level, our students work on world-class problems and their theses are published by top journals in the field. All of our doctoral students spend a substantial part of their time in well known international centers.

Outreach
Probability is now part of the mathematical program at high school and there are important needs addressed to our group. Our conviction about the importance of this problem and the role that probabilistic concepts play in modern society compels us to maintain an active series of conferences in the area.
The CMM has been a model which I have tried to emulate for the Pacific Institute of Mathematical Sciences (PIMS). On the one hand, they cover a wide range of mathematics, pure and applied; on the other, they have the best industrial connections I have ever seen. They have developed a collective structure, whereby researchers coming from pure mathematics are put in charge of projects with industrial partners. This has proven immensely successful, not only because these projects benefit from the dual efforts of academics and engineers, but also because it enhances the quality of the researchers themselves: mathematicians find new sources of inspiration in real-world problems, and are forced to broaden their knowledge of mathematics in order to understand the different facets of such problems. In addition, it has developed a strong sense of identity and cohesion within CMM, which is now recognized as one of the major centers for applied and industrial mathematics in the world, and is courted by MATHEON in Berlin, MITACS in Canada, MASCOS in Australia, and so forth.

The strong association CMM and PIMS have developed is aimed towards the mathematics of the environment. Western Canada and Chile both rely very much on natural resources, and will be hit relatively hard by climate changes. These issues are rife with scientific problems, and we aim to develop the mathematical aspects, within the framework of our joint association with CNRS.

Ivar Ekeland
Canada Research Chair in Mathematical Economics
University of British Columbia
Fragmentation in Mining: Energy and Ore size distribution in the Mining Process

One of the main goals in mining is to liberate the metal contained in mineral blocks and separate it from non-valued content. Fragmentation is carried out in a series of steps: the first one being blasting, after the material is transported to crushers, grinders, and further, to mills. At each step the particles are screened and if they are smaller than the diameter of the mesh of a grid, they are forwarded to the next step. The process finishes when the material attains a size sufficiently small for the mining purposes.

In crushers, grinders and mills the material is broken by a repetitive mechanism, finishing when the particles can go across the classifying-grid. In the intermediate steps (blasting and crushers) output sizes are known to be not optimal in terms of the global energy cost. To get an idea of the magnitudes involved in fragmentation in the mining industry, at Chuquicamata, one blasted and transported nearly 600,000 tons of material per day. Out of this, nearly 200,000 tons are defined as mineral, the rest is waste or sterile. After blasting, there are two circuits for mechanically decrease the mineral size, the conventional process uses three types of crushers and the particles are forward to the mills once they are smaller than 1/2 inches.

The fragmentation process is stopped once the particles have a size smaller than 20 mm.

One of the problems that mining industry faces is to minimize the energy used in these processes and to control the efficiency of the blasting and crushing process to optimize the fragmentation cost. Indeed, as nearer to the pit the liberation of the mineral is achieved, the better the economics of the whole operation is.

The measurement of the efficiency of blasting and crushing is crucial to optimize the breakage energy of the mining process. Various techniques are used to characterize the size distribution produced by blasting and crushing operations being Image Analysis the most used. Automated picture has the weakness of the statistical uncertainty of its results because what is seen is only the superficial layer. The natural segregation produced by the mass of the particles and its size distribution at the post blasting hip, the conveyor belts movement and the limits of the photo are distorting the real granulometry size distribution.

The aim of our program is to describe the fragmenting process subjected to induced energy. A main theoretical result is our estimation of the energy E(a) which is needed to reduce a unit mass to fragments of size at most a, when is small (details in Fragmentation Energy. Adv. Appl. Probab. 37 (2005)). The use of this result in realistic problems is under studied. Also several Mathematical Engineering Theses, in collaboration with engineers and scientists of IM2, are devoted to improve the actual techniques of image analysis used in granulometry.

Rockmass Geo-Mechanical Instabilities Induced by Excavations in El Teniente: a codelco copper mine

El Teniente is one of the largest underground copper mine in the world. It is located in the first elevations of the Andes mountain chain in the central zone of Chile, about 70 Km SSW from Santiago. Its copper orebody has two very different forms: the secondary ore is located near the surface while the primary ore is deeper and consists of high cohesion and impermeable rockmass with much harder geo-mechanical behavior than the secondary ore.

The exploitation of the primary ore started in the beginning of the 1980’s. The caving method is initiated by the blasting of the bottom volume of the rockmass column. The broken material is then mined
out creating cavities that allow the gravity force to continue the fracturing process. The subsequent extraction of new broken material ensures the continuity of the process, propagating the rockmass fractures to upper levels. These excavations induce elastic and later inelastic deformations within the surrounding rockmass. Part of the stored potential energy is gradually released during the process of inelastic deformation without any consequence on the production. However, under some circumstances a significant amount of energy may be released by a sudden inelastic deformation, radiating detectable seismic waves.

High magnitude seismic events produce damages to the surrounding excavations, being rockburst the worst of them. Indeed, serious damages have already forced the interruption of the mining operations in some production sectors.

The Department of Mine Planning at El Teniente initiated at the beginning of the 1990’s a systematic study of the rockbursting phenomenon. A digital seismic monitoring system was installed in 1992 covering the whole mine with 25 seismic stations. This network locates and quantifies seismic sources with very good precision. Since the installation of this system, almost 140,000 events have been recorded (location, seismic moment, radiated energy, magnitude). This huge amount of data must be previously classified and statistically analyzed before being of any use. Although some methodologies have been developed in order to reduce the rockbursting risk, these are not satisfactory for the mining plan in the short term. Thus, the processes responsible for the complex dynamics associated with caving methods are still far from being well understood.

The main concern of this on going project is the development of techniques to predict, locate and eventually prevent potential rockmass instabilities that may produce rockbursting. The control variables in this
process are some mining parameters such as geometries of excavations, excavations sequences and extraction rates. One of our main objectives is to perform a mathematical modeling and analysis of the dynamical redistribution of stresses/deformations within the rockmass, taking into account the geo-mechanical properties of the rockmass and the most relevant dynamic aspects of the problem. The aim is to locate potential instabilities that may induce high magnitude seismic events and occasional rockbursts. In doing so, we expect to develop quantitative and qualitative mathematical tools for the determination of some mining parameters such as excavation geometries and extraction rates in order to design more economically efficient and safe mining plans.

Models for the Dynamic Interfaces in the Fusion, Conversion and Refinement of Copper

The principal technological impact of this project is the improvement of the fusion of copper concentrate at the Teniente-converter via cost reductions in production and an improvement in efficiency.

We have a better understanding of the multiple processes and the physical-chemical phenomena involved in copper metallurgy. Mathematical models formulated for analysis and simulation provide the basic tools for research and the starting point for all the studies of this project. The theory of nonlinear partial equations and other numerical methods are used for theoretical analysis and computer simulation. However, it is well known that this type of approach alone is insufficient, and it must be complemented with physical models that produce experimental data to validate the results generated by computer simulation.

The existence of an interdisciplinary group of researchers at CMM and IM2 makes this a propitious opportunity to carry out a project of this nature and scope. Thus, the topics involved here will take up a central position in the tasks of this group.

This study proposes to optimize and improve the efficiency of the processes of fusion, i.e. conversion of copper in the Teniente-converter, from the perspective of the distinct phases of the processes. This will involve introducing operational modifications and design changes while maintaining the perspective that new continuous processes of conversion may occur over a brief period. Areas of focus include: control and monitoring of wave motion induced by bubble injection or by blowing through nozzles, formation and growth of accretion in the nozzles, formation of the magnetite layer, and cold-load fusion. Technological innovation in these areas will contribute to reducing operational costs, diminishing emissions, and improving working conditions in the foundries of copper concentrate.

Specific objectives

– Improve the availability of equipment and the duration of operations at the Teniente-converter.
– Contribute to reducing operational costs, increasing productivity and improving the quality of products and working conditions at the copper concentrate foundries of CODELCO.
– Understand and optimize the efficiency of pyrometallurgical processes, in which the dynamic of multi-phase processes is most significant, introducing operational and design improvements.
– Improve technical-scientific know-how in the dynamic of multi-phase phenomena in the processes of fusion-conversion from the perspective of fluid-dynamics and material science.

These objectives will be achieved as a result of transfer of technology, based on the new knowledge developed: an improvement in the different processes of fusion-conversion will be a direct result of the understanding of the dynamic of multi-phase phenomena that play an essential role in the majority of high temperature reactors.
Reconstruction of a bacterial metabolic regulatory network with applications in bioleaching.
The technological advances that have signaled developments in biology and biotechnology during the last two decades—in particular, the speed of advances in techniques for massive extraction of biological data heralded by the sequencing of entire genomes, experiments using micro and macro-arrays for quantifying gene expression and new techniques in metabolomics—are generating such enormous quantities of information as to require the development of new computational tools and mathematical theories to handle their treatment and analysis. The size of this challenge can be measured by the fact that NSF gives very high priority for research in this area. The vision arising from mathematical modeling establishes a new approach for understanding the organization of biological information in a network. The mathematical challenge involves the creation of tools for the extraction and integration of data, models, analysis and simulations to facilitate the ready interpretation of complex networks of biological interaction.

During the last five years, the CMM has set up a program in systems biology oriented towards the biomining of copper. This has been achieved by means of the Laboratory of Bioinformatics and Mathematics of the Genome (LBMG) which is a multidisciplinary group made up of researchers, engineers and undergraduate and doctoral students in mathematics, statistics, informatics and biotechnology. Since its creation, the LBMG was associated to the Millennium Nucleus Information and Randomness and has been the reference laboratory on subjects of bioinformatics and mathematical modeling for the Chilean-Japanese company BioSigma S.A. whose mission is to be world leader in the area of biotechnology for mining. Furthermore, the LBMG has managed a FONDECYT project in systems biology since 2006. Today the LBMG is a national authority on the in silico analysis and modeling of genomic information for biomining. Since 2003, the LBMG has obtained theoretical and applied results in four areas:

- Systems for identifying biological systems or “metagenomics”: We have developed a sophisticated algorithm for the design of “oligo-chips” and primers that enable the identification of complex bacterial communities in their environment and their function. We have applied for patents in Chile, USA, France, Australia, Peru and Brazil. These

Below right: José Luis González, Oliver Langeard, Pablo Moreno, Álvaro Graves, Juan Ugalde and Andrés Aravena.

Bacterial genome annotation diagram. The LBMG has already annotated three complete bacteria.
methods are already part of the technology used by BioSigma S.A. As a result of these developments, LBMG has been awarded a Fundación COPEC-Universidad Católica project to carry out metagenomics on pollutant communities in the poultry and wine-producing industries.

Sequencing and analysis of bacterial genomes: We have developed mathematical and computational methods for assembly of genomes. As a result of this effort, we have generated sequences for the three most useful bacteria for biomining: Wenelen, Yagan and Licanantay. At the same time, the mathematical methods developed and the problems surrounding them have given rise to scientific publications in the areas of theoretical informatics and applied probability.

Functional annotation of genomes and modeling of biological networks: Here, we have been primarily concerned with annotating the three genomes we have sequenced, along with determining the networks of biological interaction relevant to the biomining process. We have given special emphasis to the development of mathematical and bioinformatic methods for detecting binding sites for transcription factors and other regulatory elements, as well as the in silico generation of networks for gene regulation and metabolic networks.

Statistical analysis of genomic data: This line of work is centered around the development of statistical methodologies for the interpretation of gene expression experiments based on micro-arrays, their posterior incorporation into networks of biological interaction and the discovery of relations in sequenced genomes. In particular, we have successfully used CART to find new target genes for the Wnt signal pathway which is important in understanding Alzheimer's disease.

Apart from the aforementioned activities, it has been necessary to create software and databases for the management and integration of genomic information in a world-class high-performance computing environment. Toward this end, we have made an agreement with U. of Bielefeld for the development of the GenDB system and we have made progress in integrating this system with platforms for simulating biological networks.

An ongoing and important activity in the LBMG is the maintenance of a high standard of staff training and international collaboration. In 2004, our laboratory organized the CMMPA School on mathematical and computational methods in biology which was attended by more than 100 students from Chile and Latin America. Six engineers who previously worked in the LBMG have gone on to pursue doctorates in bioinformatics in France and USA. We have also welcomed over 15 Chilean and foreign students to undertake their undergraduate thesis studies with us.

The principal challenge in the future for the LBMG is to consolidate our leadership in the mathematical modeling of genomic information in large-scale experiments aimed at solving problems in biotechnology and human health. In particular, some topics of interest are to develop mathematical and computational methods to: (1) decipher the regulatory signals in a genome, (2) integrate and visualize large-scale-omics information, (3) reconstruct models of gene-interaction networks and metabolic networks in order to produce a virtual cell and (4) study the shared dynamics of said interaction networks and generate hierarchical time and space models.

Reconstruction of a bacterian metabolic and genic regulatory network.
The telecommunications sector has a great deal of importance in Chile. This is due in part to the significant growth that the mobile phone market has experienced over the past few years. These apparatuses have become key tools for promoting our country’s development. In view of the fact that this sector forms part of an industry that develops its products on networks and operates in a monopolistic fashion in some areas, every five years the Ministry of Telecommunications regulates and sets rates for some of the services that these companies provide. These include the access charge tariff, or the payments that must be made to mobile phone concessionaires for the use of their networks.

**Goals**

In some areas of Chile the telecommunication market is dominated by one company that enjoys a high level of penetration. As a result, the adequate setting of rates for the services provided by the companies that participate in this market should reflect the cost of offering these services. This ensures that this activity is a lucrative one and that there are no distortions in the rates.

The Ministry of Telecommunications held a bidding process and awarded a contract for creating efficient models that allow for the setting of rates for the services provided by landline and mobile phone companies. We have grouped a multidisciplinary group formed by specialists in the fields of Economics, Mathematics and Telecommunications that has developed two efficient business models (one for mobile phone services and one for landline services). These models allow for the support of the tariffication processes developed by Ministry of Telecommunication and the companies that work in public telephone services. These optimization and equilibrium-type models allow us to obtain all rates for the services that are subject to regulation.

**Accomplishments**

We improved the processes that the government uses in order to make decisions in this area, particularly in regard to economic regulation. The introduction of more realistic and complex models led to better definitions of the variables involved and the introduction of transparency in the setting of economic policy. Various complexities have been resolved for these models, including the optimization aspect of this process, which is related to the optimal lowest cost-design of the networks (for both mobile phones and landlines). The optimal design of networks involves aspects such as the location, capacity and quantity of transmission antennae, commutation and transmission equipment and fiber-optic interconnection networks. The equilibrium part of this process involves the calculation of regulated rates that allow the companies to ensure adequate profit margins. In both cases we have generated new models, calculation algorithms and processes for their implementation and validation. These models are currently being used in the Ministry of Telecommunication and have led us to engage in the mathematical analysis of new questions in the area of Optimization and Equilibrium.
The Ministries of Telecommunications in Peru, Colombia, Venezuela and the United States have shown an interest in this development. In the case of the first two countries, the group has carried out training activities in the formulation and solution of these types of optimization-equilibrium models. Our experience with localization problems had already been published.

Relationships with Other Institutions
We have formed a multidisciplinary group composed of specialists from the Universidad de Chile’s Department of Economic and Industrial Engineering, the Universidad Católica’s Department of Electrical Engineering and the consulting firm Consultec. A regulatory agency for the area of telecommunications from each of the countries mentioned above has shown an interest in the development made by the CMM in this area.

Detail of a diagram of efficient business model for landline services.
Our interest is the analysis of transportation networks within a large city, in order to provide a stronger scientific ground to assist the decisions made by the competent authorities. In addition, several problems in transportation science are still on the way of attaining full maturity concerning its correct mathematical formulation and solution.

Equilibrium and Congestion in Public Transportation Networks
The goal here is to model the equilibrium state on a static transit network, incorporating the congestion effects upon the users decisions. In contrast with private transportation networks, the effects of congestion upon public transportation systems have until very recently been essentially neglected or introduced in a very unsatisfactory and mathematically incorrect way. This raises substantial doubts concerning the validity of using such models to describe the situation of highly congested networks as in most large cities. We fill this gap by focusing on a hyper-path formulation of the equilibrium, where we fully incorporate congestion by means of different approximations based on queuing theory. We have proved existence of equilibrium, and derived different characterizations. The latter has enabled us to devise an algorithm for computing an equilibrium, which has been tested on real-world networks. At the same time we have studied the congestion processes occurring at bus stops, using both simulation and queuing theory, in order to provide a more realistic treatment of congestion.

Algorithms for Computing User Equilibrium in Networks
User equilibrium models are used to describe the equilibrium state on a multiple origin-destination network in which travel time along each arc is flow-dependent. The standard approach to solve this problem is to directly apply Wardrop's conditions and to transform them into an equivalent convex minimization problem.

Approach based on a combinatorial decomposition of the network into regions to which Wardrop's conditions are applied by switching flows. This local approach iteratively solves the network user equilibrium and has been shown to be convergent in the case of cyclic networks. From a computational point of view, the method has been tested on real-world instances comparing favorably with respect to the more classical methods.

Urban Land-use Equilibrium Models
The spatial distribution of activities in the urban context is the result of a large number of individual location decisions taken by agents (households and firms). Each individual makes an optimal choice considering the available location set, his/her preferences and the constraints (budget and time). Location choices are different because the built environment is heterogeneous, including here the distribution of activities itself plus the accessibility provided by the available transport system, but also because...
of the diversity of agents preferences and buildings available at each location. Urban economics assumes that location equilibrium requires that each location is assigned to that agent which submits the highest bid. Suppliers decide which new land and the type of building to develop as to maximize profit given the maximum bidder rule. All agents take decisions simultaneously with complete information about the decisions been made by others. The existence and sensitivity analysis of an equilibrium for this market is a complex problem because: The equilibrium is achieved by the adjustment of rents for each location; demand is the result of a bid-auction process for each building-location option; supply follows a profit maximizing behavior; equilibrium is attained by the simultaneous satisfaction of the constraints.

Routing on Networks with Server Latency
This subject is concerned with routing on a communication/transportation network on which links may be traversed in either direction but there is a latency period each time one switches direction. Assuming that requests arrive to both link ends under a known pattern, the question is to devise a routing strategy that optimizes a given efficiency measure such as total or maximum queue length, average or maximum waiting time, throughput, etc. This occurs for instance in the case of a one-lane road that must be shared for traffic in both directions. Our study has focused on analyzing a feasibility question, namely, does there exist a control policy capable of routing the flow and keeping the waiting queues at nodes of bounded length? Different situations have been analyzed, depending on the amount of information that the controllers placed on each node may exchange.

Dynamic and Continuous Models for Traffic Flow
Since it is fairly obvious that flow and congestion is an inherently dynamic process, we started a research activity in the form of an internal seminar to explore alternative ways of incorporating a dynamic dimension to the network equilibrium models. Several models have been proposed for the dynamics of traffic flow along a route, in terms of PDE’s describing the space/time evolution of the state variables (velocity, density and flow). The goal is to understand the evolution of shock waves within the traffic. Because of their complexity, the use of these models has been essentially restricted to modeling an isolated route segment, while the effect of the rest of the system is included through the boundary conditions.
Our interest in forestry was sparked at the beginning of the 90’s at DIM. At the time, our group together with forestry specialists studied the European pine shoot moth plague which affects radiata pine plantations in Chile. Initially, we were interested in the temporal and spatial displacement of this plague and later we investigated the economic damage it causes in said plantations.

In 2001, the Forest Laboratory of the CMM began a FONDECYT project in conjunction with Fundación Chile, Pontificia Universidad Católica de Chile y el Center de Coopération Internationale en Recherche Agronomique pour le Développement (CIRAD) in France. Through this initiative, we developed a part of the simulator INSIGNI plus, for individual growth of radiata pine trees. This complements the former simulator for growth of radiata stands produced by Fundación Chile. In this project, the Forest Laboratory provided models for predicting the future quality of wood inside a tree. This enables the early detection of deformations that will still be present at the time of harvest (typically 20 years later). Young trees affected by such deformations could be extracted early on which is beneficial to the remaining trees in the plantation in terms of competition for light and nutrients.

Besides this, our laboratory has carried out basic research and development of models, principally on the radiata pine, for the evolution of trunk deformations and models for structural recovery based on hormonal regulation in trees vis-à-vis changes in the natural vertical orientation. One of our most important achievements has been the successful patenting of a metallic instrument we invented for making field measurements of trees. The measurements taken using our instrument can then be used to estimate the spatial position of sections of the tree and hence predict the future evolution of damage present in the trunk. On the top of this we have published our theoretical and applied models in the best journals in the area of forestry and mathematical biology.

Currently, we are continuing to enhance the model for structural recovery by making improvements to the graphical user interface of the software, by employing better and finer grids for approximating the tree shape and by perfecting the visualization of growth in volume. We are also working on modeling optimal investment strategies for the management and control of forest fires and analyzing remotely sensed data (mainly via satellite) to discover patterns of attack and spread of plagues in plantations.

The great challenge facing us in the future is to bring our knowledge and model development to other areas in forestry. Toward this end and keeping the need for ongoing research into native forests we have instigated the creation of a center concerned with the technological aspects of native forests. This unit, called the Technological Center for Sustainable Native Forests (CTANS), was established in conjunction with the Faculty of Forestry Science at our university, the universities of La Frontera, Los Lagos and Magallanes and researchers from well known institutions in Sweden and Finland. The CTANS is a multidisciplinary organization based on technology and mathematical modeling. It is responsible for a project whose aim is to protect native forests by understanding the role they play in areas ranging from the ecological to the social.

As was the case in Sweden, Finland and Canada, timber from native forests ought to play an important role in the development of the Chilean economy. These countries have utilized their forests in a sustainable way, generating enormous industrial expansion which is sufficient to support more than 100,000 direct jobs. CTANS will fill a need in basic research as well as in applied research on our native forests.
Expected Impacts

- To stimulate and develop high-quality research on native forests in a variety of disciplines (such as mathematics, transport, forestry, the logging industry and construction, etc.) involved in the setting of responsible and sustainable use.
- To update knowledge about this renewable resource and its relationship to the social, local and regional environment.
- The development of native forests will have as an immediate consequence the generation of a large number of jobs in the south of Chile. This will pave the way to improve the quality of life, not only for the inhabitants of these forests, but also for those who make their living from them.
- To generate projects to support industry and establish new lines of applied research and training in this area.
- To drive new lines of research on the unique biodiversity that our native forests have to offer and which holds great interest to the European scientific community.

A 3D measuring device. Each red band indicates the wood disc position to obtain a 3D spatial reconstruction for the external and internal shape of a log.

The spatial position of wood discs is determined by three points on the discs. A 3D graphical system permits the localization of points and finally a 3D image of the log.
Environmental Modeling with Emphasis on the Dispersion of Atmospheric Tracers and Their Impacts

Over the last two decades, atmospheric and climate models have evolved from isolated, decoupled, over-simplified codes to modular, coupled, complex codes. These developments have been further enhanced by the extraordinary growth in observational capabilities of the climate system and hence by the extraordinary growth in data amounts and data analyses requirements.

Over the last 10 to 15 years, Chile has faced an unprecedented economical growth and development. On the one hand, this has lead to fast technological advances and increasing consumption of resources and energy. On the other hand, the answers and actions required to make this development environmentally sustainable have become more complex and demanding. In particular, sophisticated environmental systems for weather and climate prediction, including biogeochemical aspects, are needed and requested from policy makers, industry, citizens, etc. These systems are required to address key-issues such as future availability of water resources for human consumption and energy production, air pollution potential and health risks in growing urban centers. Of course, applied mathematics play a key-role when developing and applying such sophisticated environmental modeling systems.

We aim at establishing a bridge between biogeochemical science and applied mathematics by promoting the use and application of advanced mathematics in atmospheric and climate modeling. This implies identifying subjects that require or gain from the application of mathematical results and techniques that are also challenging from a mathematical point of view. One such subject is inverse modeling related to the identification of air pollution sources and data assimilation for atmospheric chemistry-transport-deposition models, particularly those to be used in operational air quality forecast. Thus far we have engaged in several projects in order to investigate the use of various inverse techniques (e.g., optimal control approaches with weighted least squares error functional and adjoint techniques; estimators based on Kalman filters or simply best linear unbiased estimators, projection methods) to determine source strength and location for point sources of arsenic and oxidized sulfur associated with copper smelters, and diurnal and weekly variation of traffic emissions in South American mega-cities.

We are now expanding the use of these techniques into operational air quality forecast, an endeavor that will feature the collaboration of the Chilean Meteorological Office. Also, we address the analysis of spectral data from remote sensing using inverse techniques specific for the radiative transfer equations, aiming at providing methods and data for validating and correcting atmospheric models (data assimilation) and, on the other hand, validating satellite retrievals. Furthermore, in the area of regional climate modeling we will focus on the role of megacities as global change drivers and the coupling between the ocean and the atmosphere in the climate system. This will be done within the framework of national and international projects aimed at transferring and establishing front line parallel and distributed computing technology for chemical weather forecast and regional climate modeling for our region for both scientific and operational use.

South American Emissions Megacities and Climate

SAEMC 2006-2010 is a four year international project financed by the Inter American Institute for Global Change (IAI) in which five countries, 20 researchers and 12 postgraduate students participate from 9 institutions.

Objectives

To provide accurate regional emissions and climate change scenarios for South America, with emphasis on the impacts of and on megacities.
To establish the basis for operational chemical weather forecast for South American megacities.

To strengthen and expand an active research and capacity building network in the Americas functional to Earth System Modeling.

Working-groups

– Mobile and Stationary emissions scenarios estimate and evaluation.
– Dynamical down-scaling of climate change scenarios.
– Pilot implementation of chemical weather forecast network and tools for South American megacities.
– Prospective characterization of aerosols in and downwind from South American megacities.
– Information and Technology. These areas complement each other, and as integrated they establish a key component for Earth System Modeling in the Americas.

SAEMC Workshops

The outcome of the June SAEMC 2007 Workshop was intended to the establishment of regional capabilities to address the enormous challenge of air quality degradation in South American megacities which offers an opportunity to improve and enhance hitherto disperse research capabilities and to produce scientifically sound and policy-relevant knowledge and tools. The Second SAEMC Workshop will be held in Brazil in 2008.

On-going Projects


Oxidized sulfur distribution over Northern Chile and Southern Peru for July 26th 2000. Cross section over 22.5 S.
The last couple of years we have developed two main research areas: The first one is concerned with agent behavior and decision-maker modeling in a power market system; the second one is related with how new technologies, like facts devices and distributed generation, change current planning and operation models. All these issues have required new mathematical model developments, algorithms and implementation techniques due to the optimization/uncertainty behavior of the agents participating in this system, the huge number of variables, states and the data bases involved and the new technical characteristics of the devices, among others components. Some of these developments have been already implemented in an ‘oriented object system’.

**Power Market Systems**

In order to attack different kinds of market equilibrium problems we have adopted an approach which combines stochastic optimization and Nash equilibrium models. Uncertainties come mainly from hydro components of the power generation big reservoirs, future electricity demands and prices. Nash equilibrium behavior comes from the fact that usually few economics agents participate in this market sharing a network transmission and the decision markers (generating companies) have individual profit [unctions and private information. Thus, the whole model is a Nash Bayesian or Cournot-Nash equilibrium problem where the computation of each payoff (profit company) requires the solution of a huge multi-period stochastic programming problem. We have two versions, the first one considers a centralized dispatch or transmission (close to the Chilean case) and the second one transmission is decentralized (British or American System). We have studied these models from the analytical and algorithmic point of view: Existence of equilibrium, sensitivity analysis with respect probabilities, algorithms to compute equilibrium, decomposition techniques and parallelism. Some of these developments have been already implemented using an object oriented programming approach.

**FACTS Devices and Distributed Generation**

FACTS (Flexible AC Transmission Systems) involve several families of alternating current transmission systems incorporating power electronics-based and other static controllers to enhance controllability and power transfer capability. Distributed generation corresponds to small electric power source connected directly to the distribution network or on the customer side of the meter (photovoltaic, wind, fuel cells, microturbines). This refers to generation units located near a consumption point so that power can be delivered without making use of the transmission grid. That injects power directly in the distribution network near to load. The group formulates an Optimal Power Flow (OPF) Model that incorporates these technologies explicitly.

We have studied the convergence of numerical methods for non-linear programming based on different numerical variations of the scp technique. The development of a specialized scp model for the cis is highly interest.

**Relations with Power Generation Companies**

The private companies and government institution which have been collaborating with our group are ENERGIS, COLBUS and the telecommunication secretary bureau throughout discussions, seminars, information of the system, supporting students.
Modurban: Modular Urban Guided Rail Systems

This project is developed in the context of the VI Framework Program of the European Union, by a consortium of European institutions, including system manufacturers (Alstom, Siemens and Bombardier), subsystems suppliers, operators (RATP, Metro-Madrid and Metro-Barcelona), industrial associations and academic partners.

The main target of this project is to design, develop and test open common core system architecture and its key interfaces, paving the way for next generations of urban-guided public transport systems. This approach will be applied to new lines as well as the renewal and extension of existing lines.

Our main task is to model the system of heat, ventilation and air conditioning in a total system approach and develop a simulation/optimization software tool, to reduce energy consumption in a Metro network and offering, at the same time, the best passenger comfort in vehicles, platforms and stations.
The possibility of visual access to the interior of the human body without surgical intervention and the observation of structural, functional and metabolic aspects of organs and systems is having a tremendous effect on the diagnosis, treatment, and follow up of diseases.

One important problem is the comparison of images obtained through different procedures and types of signals, each one providing its own information content while complementing the rest. This comparison produces new information with respect to the illness and is useful in guiding the surgeon or specialist to make their diagnostic or treatment interventions with improved precision.

There is much clinical experience to be gained in correlating images that provide structural, functional or metabolic information (Radiology, Magnetic Resonance, Ultrasound, Nuclear Medicine) with a variety of applications. Furthermore, sophisticated mathematical algorithms are required for a variety of methods from visual analysis of analog images to the comparison of digital images.

**Description of the Study**

This project investigates and develops mathematical-computational technology for diagnosis and surgical support guided by images. The specific applications presented below were developed based on their feasibility, simplicity, and clinical relevance, taking into account the interests and needs of the participating institutions, the originality of the research and its potential for expansion into other areas which would increase its impact in the Health sector.

- Fusion of SPECT and US 3D images for diagnosis of coronary cardio-pathology.
- SPECT and 6D localizer based system for the transeptal puncture in hemodynamic procedure.

In the first two problems, the doctor makes a diagnosis by comparing the two different modalities of images, mentally fusing them. The purpose of this project is to improve this mental comparison using a mathematical model and software that will allow the optimized representations of the fused images to be visualized on a screen.

The transeptal puncture, corresponding to the third problem above, is necessary in the treatment of patients who suffer cardiac pathology and present a very high risk of complications for doing this procedure blindly. Currently this procedure consists of moving a catheter with a needle from the femoral vein to the heart and crossing a 2cm zone of septum, called the fosa ovalis. This project proposes to develop a passive and internal follow through of the catheter towards the fosa ovalis.

The working team is composed of a dozen researchers from diverse areas including: mathematical modeling (CMM), clinical medicine (Hospital San Juan de Dios), electrical engineering and information technology for imaging (TMC and Department of Electrical Engineering) and an important collaboration with Praxim (France).

Finally, this project contributes to the training of qualified personnel at the graduate and undergraduate levels with the aim of establishing a business unit for the development and commercialization of this technology in Chile. This unit would be associated with Praxim and linked with Chilean and French academic Centers that carry out research in these areas.
In Chile, the management of natural resources has a great impact on its economy. For this reason, we have branched out into the management of fishing and microbial ecology. The people who work in this area form an interdisciplinary group of researchers who come from different academic and governmental units throughout the country and overseas, including Peru and France. Using mathematical tools such as control theory, optimal control, game theory, viability and models from optimization and statistics, we are able to obtain a more complete understanding of how to determine regulations, negotiations and policies of extraction, while at the same time considering the effects of political, biological, climatic and economic changes in the area. We highlight the following modeling efforts:

### Interaction Between Agents Exploiting the Same Fish Resource

The law regulating national fishing enterprises defines exclusive fishing rights to the traditional fishing fleet when it operates within the first 5 nautical miles. The commercial fishing fleet does not have access to this zone. These two kinds of agent undertake exploitation on very different scales, both geographically and in terms of the volume of catch taken. Their interaction ought to be such that fish transit between the two zones associated with the agents although the patterns of migration are not completely understood. Our aim here is to use game theory to simulate this interaction.

### Study of New Indicators for Sustainable Fish Exploitation

Using mathematical techniques for the controllability of systems in discrete time, we can study when a fish exploitation policy is viable. This is when a desirable configuration, for example, from the economic point of view, is sustainable over time. In particular, we want to study if the biological indicators currently used are effective for these purposes. Further, we will study other possible indicators that can be used in the future to achieve the sustainable exploitation of fish resources.

### Optimal Control of Sequential Batch Bioreactor for Waste-Water Treatments

Sequential batch reactors are often used in biotechnological industries, notably in waste-water treatments. Typically, a tank is filled with activated sludge or biological micro-organisms capable to degrade some undesirable substrate. The method consists then in a sequence of cycles controlled by a user that intends to manage efficiently the whole process, that is, in a minimal time. The time necessary to achieve such cycles can be substantially long and have economical impact on the overall process. Hence, manipulating these cycles has clearly an influence on total duration of the process. But the nonlinear kinetics of the biological reactions does not make always obvious to determine which strategy minimizes its total time. We thus study optimal control and optimal design problems for finding this strategy and other related issues.
The CMM is strongly involved in information security in collaboration with the Department of Computer Science (DCC) at the Universidad de Chile. Among our most outstanding achievements, we highlight the following:

**CLCERT**
The Laboratory of Applied Cryptology and Security (CASLab) came into existence in October 2001 as a long-term joint project between the CMM and the DCC. In 2006, this laboratory became a virtual network of researchers with a common interest in information security, particularly in applied cryptography. To gain visibility in the local security scene, the Chilean Computer Security Incident Response Group (CLCERT) was formed.

In matters of information security, the CLCERT vigorously endeavors to educate and raise the awareness of the general public. This is achieved through the distribution of documents containing advice for final users, alerts, news, security tips, editorials and technical reports, etc., all of which are published on its Web site. Because of the wealth of information it makes available, the CLCERT’s Web portal has become the most important on-line resource in the country in the area of computer security, receiving more than 15,000 visits per month originating from almost 7,000 different sites.

Since the beginning of 2004, the CLCERT has funded itself by means of the training it offers and more recently via alliances it has formed with important national companies such as Microsoft-Chile which are interested in supporting its activities nationwide. Other key work conducted by the CLCERT is in the creation of public policy concerning the security of information systems. This work is most clearly reflected in the wording of two laws, 83/2005 and 93/2006, which have recently been adopted. The professional and academic prestige of the CLCERT, together with its technological impartiality and neutrality, were determining factors in producing these laws. A direct consequence of Law 83/2005 was the designation of security officials in each of the State’s administrative organs (around 1,000 officials throughout the country).

Recently, the CLCERT has been invited to join the government committee overseeing the implementation of Law 83/2005.

**Electronic Invoices**
The electronic (or digital) invoice is a means of supporting commercial transactions between taxpayers that replaces paper invoices. The electronic invoice is a key initiative in simplifying Chile’s taxation system and will save the country an estimated US$300 million per year. The CLCERT participated in the design of this system together with the DCC, NIC Chile and Internal Revenue Service.
The concept of e-Science plays an important role in world-class research. For this reason, the CMM has delved into the field of high-performance computing (HPC) since 2005. This initiative was mainly driven by our needs in genomics, climate, transport and energy. With a Fundación Andes’ grant we purchased a cluster comprising 32 Itanium2 processors. Later, a donation of Infiniband technology by Hewlett-Packard dramatically increased the general performance of the system. On the other hand, our close cooperation with the San Diego Super Computing Center and the cluster software development team enables us to quickly acquire know-how and expertise.

So far the cluster has served as a platform for developing and improving some of our applications. For example, the LBMG, working with BioSigma, used it for the mass annotation of genomic sequences that serve in evaluating the output of bacteria in the bioleaching process employed by CODELCO. The current vision of CMM is to consolidate the laboratory as a South American authority on HPC and Grid Computing.

To achieve this objective, we are pursuing the following activities:

**SAEMC Project**
We are working with the Chilean Bureau of Meteorology (DMC) with the aim of modernizing its cluster and improving its climate forecasting model. The DMC and CMM clusters stay connected using Grid technologies, integrating the two platforms and allowing the DMC to give new operational forecasts such as air quality.

**CLGrid**
The CMM is leading a national initiative for Grid Computing. In order to accomplish this, we have formed alliances with the main universities in the country. To spread the CMM knowledge the Lab has done several workshops on Grid Computing where the cluster group demonstrated its expertise on e-Science.

**Hewlett-Packard and CLGrid**
HP has found a niche business in CLGrid and supports this initiative by enabling our engineers to become qualified at HP Labs in San José, California. HP is positioning the CMM to be the perfect ally for commencing e-Science projects in the private sector.

**GELATO**
HP has pushed for a federation concerned with the motivation and exploitation of the capabilities of the Itanium2 processor, both in research and business. This federation is composed of several universities around the world, including the Universidad de Chile. We have attended three international workshops where the CMM’s work in this area has been exhibited.

**PRAGMA**
The CMM is the only operating node of PRAGMA Grid in South America. This network is composed of countries such as Australia, Japan, China, Taiwan, the United States, Mexico and Chile. Finally, we have cooperated with various companies and institutions in evaluating the use of HPC in their projects and research. These include Endesa, IM2, Liceo Manuel de Salas and the Departments of Mechanics, Computer Science and Geophysics at the Universidad de Chile.
The atmosphere at the CMM is a very creative one, there are lots of seminars and visitors so there are plenty of opportunities for discussing with diverse people and learn about new fields and new subjects. At the CMM I don't have to teach, it is a great opportunity for me to just concentrate in my research, just doing my work. So I decided to stay at the Center for another year.

Michael Schraudner
Universität Heidelberg, Germany
Postdoc CMM
The CMM belongs to the Faculty of Physical and Mathematical Sciences at the Universidad de Chile, which houses the most prestigious engineering school in the country. From this undoubtedly advantageous position, the CMM has a great influence at the national level over the education of engineers, doctors and postdoctorates. Besides developing a high standard of applied research, the CMM gives specialized training in mathematical modeling to engineers working for companies involved in joint projects with it.

Undergraduate Program
The engineering degrees offered in our faculty have a broad background in basic science with a two-year common plan and a solid presence of mathematics courses. On average, researchers in the CMM give 40 classes per year in the common plan with an annual throughput of approximately 3,500 students. This fact translates into a large impact on the education of new engineers, who benefit from the experience in basic and applied research that the CMM’s members have. On the other hand, the objective of the undergraduate degree program in mathematical civil engineering imparted by the Department of Mathematical Engineering (DIM) is to provide a solid grounding in applied mathematics that will enable our graduates to confront complex engineering problems by formulating and solving models using advanced mathematical tools. In recent years, enrollments in the degree have increased significantly which has resulted in an average of 25 graduations per year over the last three years. This growth can be explained by the interest that the CMM awakens in students in the faculty since it gives them a window into applications and mathematical research of the highest standard. Our engineering training takes place in a highly competitive environment and our students are not only the best in the faculty (40 points over the average 700 in a national test used to enter Chilean universities), but the best in the country.

Ph.D. Program
Our Ph.D. in mathematical modeling at Universidad de Chile—which was inaugurated in 1998—produces doctors of the highest standard who carry out original research and whose results constitute a significant contribution to our knowledge about applied mathematics. Since its beginnings, this program has experienced sustained growth and has now produced a total of 33 doctoral graduates. Currently, there are 30 Ph.D. students enrolled in the program and the graduation rate has reached 9 per year. These figures are quite high compared to what is typically found in doctoral programs in mathematics in Chile. The Ph.D. Program in applied sciences at Universidad de Concepción has produced 10 doctoral graduates and currently has 18
enrolled students. The CMM plays a fundamental role in these programs. The cooperation between DIM and CMM with institutions abroad, particularly the CNRS, generates opportunities for interaction between Ph.D. students and guest professors. Between 1999 and 2006, visiting professors have given a total of 21 courses on different subjects as part of the Ph.D. Program. This has also allowed the students to make direct contact with researchers from top international centers around the world. The CMM is also instrumental in supporting our students by organizing overseas research visits (20 during 2005-2006) and arranging attendances at overseas courses and congresses (21 in the same period). All this flows over into the quality of research work leading to theses.

**Postdoctoral Positions**
During the last three years, we have had 20 postdocs in the CMM, with an increasing number of applications (26 in 2006), demonstrating a rising interest in the CMM by young doctors around the world. Our postdocs come from Latin America (Argentina, Brazil and Chile), Europe (Germany, France, Spain, Rumania and Switzerland), the USA and other parts of the world such as Australia, China, India, Morocco and Tajikistan. The majority of them occupy now academic positions in important research centers throughout the world. For example, in Chile and Latin America: U. del Bio-Bio, Pontificia Universidad Católica de Chile, U. de Santiago, U. Técnica Federico Santa María, U. Adolfo Ibañez, U. de Chile, U. de la República (Uruguay). In Europe: U. Paris, U. de Pau, U. de Taulon, U. de Strasbourg, U. de Montpellier, U. de la Coruña, U. de la Laguna (Tenerife), U. of Bucharest; and in other parts of the world: Indian Institute of Technology, Shanghai U., College of Science (Saudi Arabia), Khorog University (Tajikistan).

**Professional Development**
In a number of medium to large projects, the CMM has worked with 16 engineers who finished their studies in the CMM along with 30 engineers employed by our industrial partners and who collaborate closely with our faculty and scientists to solve industrial problems. These professionals benefit from the research environment and world-class modeling offered by the center. In particular, we run joint seminars where we and our visitors share new developments in the areas cultivated by the CMM.

The CMM uses its computational infrastructure and library to support and sustain all of these activities. The CMM’s library has a collection of specialized journals which currently comprises 153 titles. Over the years, the library has seen a significant increase in the number of journal subscriptions and book additions. For example, the collection of books currently consists of 9,000 titles, which are up from the figure of 7,500 titles held by the library in 2000.

The math library at DIM.
The creation and reproduction of scientific knowledge is the responsibility of every research center and in the CMM, we envisage that this responsibility extends to supporting the development of mathematics in the national education system. We as society should cultivate all the talents that the children and youth of our country have to offer, so as to awaken their scientific curiosity and spur them on to greater levels of achievement. At the same time, we must give our citizens the tools for understanding a world in which science is ever more present in our daily lives. Being focused on mathematics, our center contributes to this national endeavor that will allow our country to reach higher goals in its development.

The incorporation of information technology at the various stages in the process of teaching and learning is an innovative challenge that has been instated in every aspect of education. In the CMM we have accepted this challenge with an emphasis on primary and secondary school level mathematics by successfully bringing a number of projects to fruition. The first of these was Innovative Didactic Materials for the Learning of Higher Mathematics in which we succeeded in producing educational software and teaching materials with virtual support.

The project Interactive Tutorials in Mathematics, aimed at talented high-school students, deserves special mention. This initiative presented students with the opportunity to learn some advanced secondary mathematics through virtual courses that were mainly oriented towards probability, geometry and the development of logical thinking. The results of this project were at the level of a prototype and clearly indicated the commercial and educational potential of the concept.

In order for education to succeed in raising the scientific level in the country we require that several aspects of the education process be strengthened. Clearly, the teacher plays a key role here, a point we have always perceived. So as a first step, we developed several initiatives for the continuing education of mathematics teachers, experimenting and having success with the e-learning modality in the areas of probability and geometry.

However, our preoccupation with the continuing education of mathematics teachers has gradually evolved into a concern for the initial education and training that teachers receive when starting their careers, in view of the enormous impact that this has on mass education. With this in mind, we instigated a project entitled Tools for the Training of Mathematics Teachers. The purpose of this project is to produce a collection of mathematical monographs and to introduce the use of case studies into the pedagogical training of future teachers. Led by researchers in our center, this project has a national scope and involves more than 30 academics from many universities along the country and we anticipate that it will have a significant impact on the quality of high-school mathematics teachers trained in Chile.

In a project led by our researchers and in collaboration with researchers from the Universidad de Concepción, standards were defined which precisely and exhaustively describe the mathematical knowledge future mathematics teachers ought to obtain during their initial training. Here, we have introduced a methodology at the university degree level for analyzing and describing curriculum content that is novel in the world. The CMM has contributed to the definition of educational policy at the university and national level. Researchers in our center have played a key role in the study and design of the Institute for Education at the Universidad de Chile, which has now attained status as a “Program for Research in Education”.
Also, our center continually participates in various facets of the Ministry of Education. We highlight our participation in the definition of standards for content and maps of progress in school education, the evaluation of school textbooks and a variety of advisory committees. This close collaboration also manifests in the form of direct involvement by the Ministry in projects headed by the center. Our activities in education have been enriched by collaboration with researchers and national and international centers. These academic ties have profited from the realization of events, both in Chile and abroad, on topics in education which have attracted the involvement of world-class researchers. As a product of the work we have done in recent years and the previously-mentioned collaborations and contacts, it is possible to set out some lines of scientific research in the areas of mathematical education and cognitive science around education for the coming years.
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OUR GOAL AT CMM:

TO ESTABLISH MEANINGFUL AND PRODUCTIVE RELATIONSHIPS BETWEEN ADVANCED MATHEMATICS AND ALL ENDEAVORS OF MODERN SOCIETY

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