

WORKSHOP ON NONLINEAR ANALYSIS AND CONTROL THEORY IN HONOR OF PROFESSOR

*Enrique Zuazua
for his 60th birthday*

— CONFIRMED SPEAKERS —

Fatiha Alabau • Grégoire Allaire • Giuseppe Buttazzo
Eduardo Cerpa • Jean-Michel Coron • Luz de Teresa
Miguel Escobedo • Enrique Fernández-Cara
Günter Leugering • Juan Límaco • Aurora Marica
Yves Meyer • Rafael Orive • Jaime Ortega
Axel Osses • Gustavo Perla • Alessio Porretta
Domènec Ruiz-Balet • Marius Tucsnak
Emmanuel Trélat • Mahamadi Warma • Xu Zhang



NOVEMBER 2021

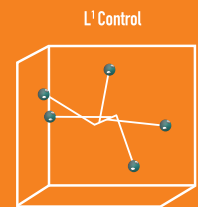
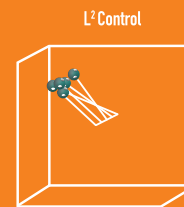
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Welcome

It is with great honor that we welcome you to the **WORKSHOP ON NONLINEAR ANALYSIS AND CONTROL THEORY** 60th Anniversary of Prof. Enrique Zuazua with will take...

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**WORKSHOP ON NONLINEAR
ANALYSIS AND CONTROL THEORY**
**In honor of Enrique Zuazua for his 60th
birthday**

November 3rd to 5th 2021 – virtual format

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Abstracts of the Talks



Workshop on Nonlinear Analysis and Control Theory in
Honor of Professor Enrique Zuazua for his 60th birthday

November 3–5, 2021 – Virtual Format

On some recent issues on the scalar-input bilinear control of PDE's

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Paris, France

Abstract

Since the early eighties, it is well-known that the scalar-input bilinear control in infinite dimensions encounters obstructions. This is closely related to the topological properties of the set of reachable states. It has a dense complement in the usual functional frame. This has first been proved for bilinear control of mechanical systems such as the beam model, and then for the control of quantum systems.

Since then, several positive controllability results were obtained with spatial localization of the control (through a real-valued potential function). These results require as a necessary condition, that an infinite number of real quantities (involving the control, the eigenstates and the targeted eigenstate), are non vanishing. It was also proved that an appropriate asymptotic behavior of these quantities is a sufficient condition for bilinear controllability results to hold. These results mainly concern dispersive equations such as the Schrödinger equation, or hyperbolic equations as the wave or beam equations.

The first purpose of this talk is to present some of the recent results on the bilinear control of first order in time evolution PDE's in an abstract framework, in view of applications to parabolic equations (joint works with P. Cannarsa and C. Urbani). The second purpose is to present a new mathematical methodology to construct explicit infinite sets of potential functions, and provide algorithms to exhibit explicit large classes of control operators that satisfy the necessary and sufficient conditions required for scalar-input controllability to hold. This gives direct concrete results for the bilinear control on heat equations, but also on Schrödinger equation, wave equations, beams equations as well as different examples of boundary conditions (joint work with C. Urbani)

Joint work with:

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Cristina Urbani², Department, University, City, Country.

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Shape optimization with imperfect interfaces

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Abstract

Motivated by the optimization of supports for structures built by additive manufacturing techniques [1], we study shape optimization problems for two different materials separated by an imperfect interface. For example, in the context of a diffusion equation, the interface, separating two phases, is imperfect in the sense that the solution is discontinuous while the normal flux is continuous and proportional to the jump of the solution. For a two-phase optimal design problem the shape derivative of an objective function with respect to the interface position is computed by the adjoint method [2]. Numerical experiments are performed with the level set method and an exact remeshing algorithm so that the interface is captured by the mesh at each optimization iteration. Comparisons with a perfect interface are discussed in the setting of optimal design or inverse problems. Some applications to support optimization for additive manufacturing will be shown.

Joint work with:

Benjamin Bogosel¹, CMAP, Ecole Polytechnique, Palaiseau, France.

Matias Godoy², CMAP, Ecole Polytechnique, Palaiseau, France.

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Variations on a theme by Cheeger

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Abstract

We present some generalizations of the well-known Cheeger inequality $4\lambda(\Omega) \geq h^2(\Omega)$, where $\lambda(\Omega)$ denotes the principal eigenvalue of the Dirichlet Laplacian and $h(\Omega)$ is the Cheeger constant. The domain Ω may vary either in the class of *all* open bounded subsets of \mathbb{R}^d or in the subclass of *convex* domains. In particular, the existence of optimal domains for the shape functional $\lambda(\Omega)h^{-2}(\Omega)$ is deeply discussed.

Joint work with:

Lorenzo Brasco, Department of Mathematics, University of Ferrara, Ferrara, Italy.

Luca Briani, Department of Mathematics, University of Pisa, Pisa, Italy.

Francesca Prinari, Department of Mathematics, University of Pisa, Pisa, Italy.

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Stabilization for the KdV equation: from Zuazua's result to the most recent ones

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Abstract

From [1] the stability of the Korteweg-de Vries (KdV) equation

$$y_t + y_x + y_{xxx} + yy_x = 0 \tag{1}$$

on a bounded domain $[0, L]$ has been largely studied. In that paper it is proved that if

$$L \notin \mathcal{N} := \left\{ 2\pi \sqrt{\frac{k^2 + kl + l^2}{3}}; k, l \in \mathbb{N}^* \right\},$$

then the origin is locally asymptotically stable in $L^2(0, L)$. If $L \in \mathcal{N}$, it is known from [2] that the linearized system around the origin admits solutions conserving its energy. Consequently different approaches, as including internal damping terms (with or without saturation) or boundary (full state or output) feedback laws, have been used to add dissipation to the system. Concerning the original nonlinear equation (1) with no feedback controls, it has been proven in a series of papers starting with [3] that for many $L \in \mathcal{N}$ the origin is still locally asymptotically stable in $L^2(0, L)$. In this talk we intend to cover these developments including this kind of questions (stability, stabilization) for systems of KdV equations with less controls than equations [4, 5].

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Obstruction to the small-time local controllability for a KdV control system

Jean-Michel Coron*
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Abstract

This talk is devoted to the local null-controllability of the nonlinear KdV equation equipped the Dirichlet boundary conditions using the Neumann boundary control on the right. Lionel Rosier proved in [5] that this KdV system is small-time locally controllable for all non-critical lengths and that the uncontrollable space of the linearized system is of finite dimension when the length is critical. Concerning critical lengths, Emmanuelle Crépeau and I proved in [3] that the same result holds when the uncontrollable space of the linearized system is of dimension 1, and later Eduardo Cerpa in [1], and then Eduardo Cerpa and Emmanuelle Crépeau in [2] established that the local controllability holds at a finite time for all other critical lengths. We present in this talk a recent result [4] showing that, for a class of critical lengths, the nonlinear KdV system is *not* small-time locally controllable.

Joint work with:

Armand Koenig¹, CEREMADE, Université Paris Dauphine-PSL, Paris, France.

Hoai-Minh Nguyen², LJLL, Sorbonne University, Paris, France.

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Instability of some equilibria in an homogeneous Boltzmann equation for radiation

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Abstract

I will describe recent results that show some instability of some equilibrium of the Boltzmann type equation commonly used to describe the Compton scattering in radiation. The linearized equation will first be considered. If time allows, remarks on the nonlinear equation will also be presented.

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Controlling some PDEs with nonlocal in space terms

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Abstract

In this talk, we will consider control systems for parabolic and hyperbolic systems where we find terms that are nonlocal in space. Our main aim is to prove controllability results with distributed controls, locally supported in space. We will deal with several different situations, starting with linear equations containing integral terms with analytical kernels, analyzed in a paper in collaboration with Qi Lü and Enrique Zuazua. In our analysis, a number of techniques will be used: sharp observability estimates, compactness-uniqueness arguments, fixed-point theorems, etc. Also, some numerical results will be exhibited. Finally, possible extensions and open problems concerning other nonlocal systems will be mentioned.

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Space-and time domain decomposition of control problems for PDEs on networks

Günter Leugering

Abstract

We consider problems of optimal control and controllability for PDEs on networked domains and discuss non overlapping space- and time domain decompositions of the corresponding optimality systems. The methods which originally go back to the classical paradigm by P.L. Lions for elliptic problems and the corresponding work by J.L. Lions result in decompositions that, in turn, can be interpreted as optimality systems for virtual control problems on a small space-time subdomain. We prove convergence of the iterative scheme and discuss a posteriori error estimates. Some numerical examples are given.



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Local null controllability of a quasi-linear parabolic equation in dimensions 2 and 3

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Abstract

In this work, we studied the null controllability in dimension 2 and 3 of the following model,

$$\begin{cases} y_t - \nabla \cdot (a(y)\nabla y) = v1_\omega, & (x, t) \in Q = \Omega \times (0, T) \\ y = 0, & (x, t) \in \Sigma = \partial\Omega \times (0, T) \\ y(0) = y_0, & x \in \Omega, \end{cases}$$

where $a : \mathbb{R} \rightarrow \mathbb{R}$ is a C^3 function that possess bounded derivatives and $0 < m < a(t) < M$.

Joint work with:

E. Fernandez-Cara¹, Dpto. EDAN, Universidad de Sevilla, Sevilla, Spain.

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November 3–5, 2021 – Virtual Format

Hardy inequalities for potentials with countable number of singularities

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Abstract

HI - Hardy inequalities for potentials with countably many singularities of the form

$$V = \sum_{k \in \mathbb{Z}} \frac{1}{|x - a_k|^2}$$

are not a trivial issue. In principle, the more singular poles are, the less the Hardy constant is: it is well-known that in all the existing results about the HI with finite number of singularities the best constants converge to 0 with the number n of singularities going to infinity.

In this talk, we provide an example of nontrivial HI in right cylinders of fixed radius $R > 0$ in \mathbb{R}^d , for a potential V defined above having the singularities $\{a_k\}_{k \in \mathbb{Z}}$ uniformly distributed on the axis of the cylinders. For this example, we prove that an upper bound for the Hardy constant is $(d-2)^2/4$, the classical Hardy constant in \mathbb{R}^d corresponding to one singular potential. We also prove positive lower bounds of the Hardy constant which allow to deduce that the asymptotic behavior as $R \rightarrow 0$ of the Hardy constant coincides with $(d-2)^2/4$. The proof of the main result lies on using a nice identity due to Allegretto and Huang [1] for particularly well chosen test functions.

Joint work with:

Cristian Cazacu¹, Faculty of Mathematics and Computer Science, University of Bucharest, Romania.

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November 3–5, 2021 – Virtual Format

Spikes and Waves

Yves Meyer

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November 5, 2021

Abstract

An atomic measure on \mathbb{R}^n is a crystalline measure if it is supported by a locally finite set and if its distributional Fourier transform is also supported by a locally finite set. Crystalline measures were introduced in 1959 by Andrew Guinand, Jean-Pierre Kahane, and Szolem Mandelbrojt. Crystalline measures play a seminal role in the following problem on Schwartz functions: One are given two locally finite sets E and F . We demand that the only Schwartz function f vanishing on E and whose Fourier transform vanishes on F be the zero function. Finally this investigation applies to the solution by Maryna Viazovska of the packing Kepler problem in dimension 8.

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November 3–5, 2021 – Virtual Format

Spectral gaps in planar waveguides.

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Abstract

We examine the band-gap structure of the spectrum of the Dirichlet and Neumann problems for the Laplace operator in a strip with periodic dense transversal perforation by identical holes of a small diameter ε . The periodicity cell contains itself a string of holes at a distance $O(\varepsilon)$ between them. Under assumptions on the symmetry of the holes, we derive and justify asymptotic formulas for the endpoints of the spectral bands as $\varepsilon \rightarrow 0$. We demonstrate that, for $\varepsilon > 0$ small enough, some spectral gaps are surely open. The position and size of the opened gaps depend on the boundary conditions, on the strip width, the perforation period, and certain integral characteristics of the holes. We formulate the spectral problem in the periodicity cell via the Floquet–Bloch–Gelfand transform. Because the dependence on the Floquet parameter the analysis of the problem in the periodicity cell requires for a serious modification of the standard justification scheme in homogenization of spectral problems.

Joint work with:

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Sergei A. Nazarov², St. Petersburg State University & Institute of Problems of Mechanical Engineering (RAS), St. Petersburg, Russia.

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Honor of Professor Enrique Zuazua for his 60th birthday

November 3–5, 2021 – Virtual Format

Some results on discrete inverse problems for partial differential equations

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Abstract

In this work, we are interested in the study of discrete inverse problems. We note that motivated by the relation of Unique Continuation Properties (UCP) with controllability and stabilization, and more recently with inverse problems, unique continuation properties have attracted a great attention from researchers. In this talk we will present some results of UCP for discrete problems, to do this, we derive a discrete quantitative propagation of smallness for discrete harmonic functions, in particular we obtain a three sphere inequality for harmonic functions. The proof of these results are based on a Carleman estimates for a finite difference approximation of Laplace operator in arbitrary dimension with boundary terms, in which the large parameter is connected to the mesh size.

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Some inverse problems in heart valve identification

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Abstract

We present an obstacle inverse problem in fluids. We analyze it from the point of view of recovering the boundary obstacle from velocity measurements and unknown pressure. First we show how to approximate the geometrical inverse problem with an equivalent resistance parameter and then we establish some logarithmic stability results using global Carleman inequalities. We include some numerical experiments that illustrates the feasibility of the resistance equivalence algorithm.

Joint work with:

Cristobal Bertoglio¹, Bernoulli Institute, University of Groningen, The Netherlands.

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Asymptotic behavior of models in elasticity and fluids, some results with E. Zuazua

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Abstract

We briefly describe important work we did between 1996 and 2014. At that time we were interested to find rates of decay of the total energy say for example the dynamical von Kármán system and the magnetoelastic system in a bounded conductive medium. We also considered the stabilization of the KdV equation with localized damping. Several other results could be found in the references below.

During his visit to South America he always has time to discuss PDEs with colleagues from the UFRJ, UFFF, IMPA, and LNCC. Our Doctoral students enjoy very much to get suggestions on their problems on their Doctoral Thesis. Thanks so much for your help on that matter Prof. Zuazua.

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Time decay of Fokker-Planck equations with confining drift

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Abstract

The convergence to equilibrium of Fokker-Planck equations with confining drift is a classical issue, starting with the basic model of the Ornstein-Uhlenbeck process. We introduce a new approach to obtain estimates on the time decay rate, which applies to both local and nonlocal diffusions [?]. This is based on duality arguments and oscillation estimates for transport-diffusion equations. The motivation and application of these results stem from the study of long time behavior and turnpike properties of solutions in mean field game theory [?].

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Constrained control in reaction-diffusion equations

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Abstract

The classical control theory for parabolic equations does not take into account the fact that specific reaction-diffusion equations make physically sense only if the state is within $[0, 1]$, in case that the solution has to model a proportion. The talk will concern the techniques developed in [1, 2, 3, 4] in order to obtain control strategies satisfying the modelling constraints. Furthermore, it will be observed, under certain criteria, the lack of controllability due to the imposition of state constraints.

Joint work with:

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On hierarchical control of parabolic equation

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Abstract

On this conference we present control problems when applying multiple strategies to control a parabolic equation.

In classical control theory, we usually find a state equation or system and one control with the mission of achieving a predetermined goal. Frequently (but not always), the goal is to minimize a cost functional in a prescribed family of admissible controls. In the last 20 years there has been an interesting effort on attaining a "controllability" objective, that is, we want to drive the solution of a system at a time T to a particular (or close to a particular) objective.

A more interesting situation arises when several (in general, conflictive or contradictory) objectives are considered. This may happen, for example, if the cost function is the sum of several terms and it is not clear how to average. It can also be expectable to have more than one control acting on the equation. In these cases, we are led to consider multi-objective control problems.

There exist several equilibrium concepts for multi-objective problems, with origin in game theory, mainly motivated by economics. Each of them determines a strategy. We will concentrate on presenting different issues of what is called a Stackelberg strategy in the particular case of a parabolic equation.

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Exponential convergence towards consensus for non-symmetric linear first-order systems in finite and infinite dimensions

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Abstract

I will first recall some results on how to achieve consensus for well known classes of systems, like the celebrated Cucker-Smale or Hegselmann-Krause models. When the systems are symmetric, convergence to consensus is classically established by proving, for instance, that the usual variance is an exponentially decreasing Lyapunov function: this is a “ L^2 theory”. When the systems are not symmetric, no L^2 theory existed until now and convergence was proved by means of a “ L^∞ theory”. In this talk I will show how to develop a L^2 theory by designing an adequately weighted variance, and how to obtain the sharp rate of exponential convergence to consensus for general finite and infinite-dimensional linear first-order consensus systems. If time allows, I will show applications in which one is interested in controlling vote behaviors in an opinion model.

Joint work with:

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Francesco Salvarani², Dipartimento di Matematica “F. Casorati”, Università degli Studi di Pavia, Pavia, Italy

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November 3–5, 2021 – Virtual Format

Reachable spaces for infinite dimensional systems

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Abstract

We consider a problem of major interest in control theory: characterizing the set of all the states which can be attained by an infinite dimensional system by using controls possibly satisfying regularity or size constraints. The study of this question is at the very heart of finite dimensional control theory, but only at its beginning in an infinite dimensional setting. In the case of systems governed by parabolic PDEs, one of the pioneering papers is co-authored by Enrique Zuazua [1] and it describes some robustness properties with respect to non local perturbations. Motivated by the results in [1]; we develop a perturbation theory for systems which are null controllable in any positive time, with applications to both linear and nonlinear problems. We finally discuss some recent advances and open questions concerning both parabolic and hyperbolic type equations.

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**Controllability properties of fractional PDEs:
What is the state of the art?**

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Abstract

We make a complete analysis of the controllability/observability properties of fractional partial differential equations. We show what is so far known and what is still unknown.

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Operator-valued, backward stochastic Riccati equations and application

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Abstract

Riccati (type) equations date back to the very early period of modern mathematics. Some particular cases were studied more than three hundred years ago by J. Bernoulli and J. Riccati. Other important contributors on Riccati equations include D. Bernoulli, L. Euler, A.-M. Legendre, J. d'Alembert and so on. In the early stage, Riccati equations were in a narrow sense, i.e., first-order ordinary differential equations with quadratic unknowns. Later on, the term Riccati equation is also used to refer to matrix or operator equations with analogous quadratic unknowns. These equations appear in many different branches in mathematics. In particular, after R. E. Kalman's seminal work, the matrix-valued Riccati equations were extensively applied to solve many control problems. In this work, we introduce an operator-valued, backward stochastic Riccati equation for a general stochastic linear quadratic control problem in infinite dimensions. Generally speaking, the well-posedness of this equation is a challenging problem, even for a suitable notion of solutions to this equation. Indeed, in the infinite dimensional setting, there exists no satisfactory stochastic integration/evolution equation theory (in the literatures) which can be employed to treat the well-posedness of such a quadratically nonlinear equation. To overcome this difficulty, we adapt our transposition solution method, which was developed in our previous works but for operator-valued, backward stochastic (linear) Lyapunov equations.

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