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Stabilization for the KdV equation: from Zuazua's result to the most recent ones

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Abstract

From [1] the stability of the Korteweg-de Vries (KdV) equation

$$y_t + y_x + y_{xxx} + yy_x = 0 \tag{1}$$

on a bounded domain $[0, L]$ has been largely studied. In that paper it is proved that if

$$L \notin \mathcal{N} := \left\{ 2\pi\sqrt{\frac{k^2+kl+l^2}{3}};\ k, l \in \mathbb{N}^* \right\},$$

then the origin is locally asymptotically stable in $L^2(0, L)$. If $L \in \mathcal{N}$, it is know from [2] that the linearized system around the origin admits solutions conserving its energy. Consequently different approaches, as including internal damping terms (with or without saturation) or boundary (full state or output) feedback laws, have been used to add dissipation to the system. Concerning the original nonlinear equation (1) with no feedback controls, it has been proven in a series of papers starting with [3] that for many $L \in \mathcal{N}$ the origin is still locally asymptotically stable in $L^2(0, L)$. In this talk we intend to cover these developments including this kind of questions (stability, stabilization) for systems of KdV equations with less controls than equations [4, 5].

References

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