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## Obstruction to the small-time local controllability for a KdV control system

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### Abstract

This talk is devoted to the local null-controllability of the nonlinear KdV equation equipped the Dirichlet boundary conditions using the Neumann boundary control on the right. Lionel Rosier proved in [5] that this KdV system is small-time locally controllable for all non-critical lengths and that the uncontrollable space of the linearized system is of finite dimension when the length is critical. Concerning critical lengths, Emmanuelle Crépeau and I proved in [3] that the same result holds when the uncontrollable space of the linearized system is of dimension 1, and later Eduardo Cerpa in [1], and then Eduardo Cerpa and Emmanuelle Crépeau in [2] established that the local controllability holds at a finite time for all other critical lengths. We present in this talk a recent result [4] showing that, for a class of critical lengths, the nonlinear KdV system is *not* small-time locally controllable.

Joint work with:

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## References

- [1] Eduardo Cerpa, *Exact controllability of a nonlinear Korteweg-de Vries equation on a critical spatial domain*, SIAM J. Control Optim. **46** (2007), no. 3, pp. 877–899.
- [2] Eduardo Cerpa and Emmanuelle Crépeau, *Boundary controllability for the nonlinear Korteweg-de Vries equation on any critical domain*, Ann. Inst. H. Poincaré Anal. Non Linéaire **26** (2009), no. 2, 457–475.
- [3] Jean-Michel Coron and Emmanuelle Crépeau, *Exact boundary controllability of a nonlinear KdV equation with critical lengths*, J. Eur. Math. Soc. (JEMS) **6** (2004), no. 3, pp. 367–398.

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- [4] JEAN-MICHEL CORON, ARMAND KOENIG, AND HOAI-MINH NGUYEN, *On the small-time local controllability of a KdV system for critical lengths*, hal-02981071.
- [5] Lionel Rosier, *Exact boundary controllability for the Korteweg-de Vries equation on a bounded domain*, ESAIM Control Optim. Calc. Var. **2** (1997), 33–55.