



Workshop on Nonlinear Analysis and Control Theory in  
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## Controlling some PDEs with nonlocal in space terms

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### Abstract

In this talk, we will consider control systems for parabolic and hyperbolic systems where we find terms that are nonlocal in space. Our main aim is to prove controllability results with distributed controls, locally supported in space. We will deal with several different situations, starting with linear equations containing integral terms with analytical kernels, analyzed in a paper in collaboration with Qi Lü and Enrique Zuazua. In our analysis, a number of techniques will be used: sharp observability estimates, compactness-uniqueness arguments, fixed-point theorems, etc. Also, some numerical results will be exhibited. Finally, possible extensions and open problems concerning other nonlocal systems will be mentioned.

### References

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