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Hardy inequalities for potentials with countable number of singularities

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Abstract

HI - Hardy inequalities for potentials with countably many singularities of the form

$$V = \sum_{k\in\mathbb{Z}} \frac{1}{|x-a_k|^2}$$

are not a trivial issue. In principle, the more singular poles are, the less the Hardy constant is: it is well-known that in all the existing results about the HI with finite number of singularities the best constants converge to 0 with the number n of singularities going to infinity.

In this talk, we provide an example of nontrivial HI in right cylinders of fixed radius $R > 0$ in $\mathbb{R}^d$, for a potential V defined above having the singularities $\{a_k\}_{k\in\mathbb{Z}}$ uniformly distributed on the axis of the cylinders. For this example, we prove that an upper bound for the Hardy constant is $(d-2)^2/4$, the clasical Hardy constant in $\mathbb{R}^d$ corresponding to one singular potential. We also prove positive lower bounds of the Hardy constant which allow to deduce that the asymptotic behavior as $R \to 0$ of the Hardy constant coincides with $(d-2)^2/4$. The proof of the main result lies on using a nice identity due to Allegretto and Huang [1] for particularly well chosen test functions.

Joint work with:
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References

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