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Operator-valued, backward stochastic Riccati equations and application

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Abstract

Riccati (type) equations date back to the very early period of modern mathematics. Some particular cases were studied more than three hundred years ago by J. Bernoulli and J. Riccati. Other important contributors on Riccati equations include D. Bernoulli, L. Euler, A.-M. Legendre, J. d'Alembert and so on. In the early stage, Riccati equations were in a narrow sense, i.e., first-order ordinary differential equations with quadratic unknowns. Later on, the term Riccati equation is also used to refer to matrix or operator equations with analogous quadratic unknowns. These equations appear in many different branches in mathematics. In particular, after R. E. Kalman's seminal work, the matrix-valued Riccati equations were extensively applied to solve many control problems. In this work, we introduce an operator-valued, backward stochastic Riccati equation for a general stochastic linear quadratic control problem in infinite dimensions. Generally speaking, the well-posedness of this equation is a challenging problem, even for a suitable notion of solutions to this equation. Indeed, in the infinite dimensional setting, there exists no satisfactory stochastic integration/evolution equation theory (in the literatures) which can be employed to treat the well-posedness of such a quadratically nonlinear equation. To overcome this difficulty, we adapt our transposition solution method, which was developed in our previous works but for operator-valued, backward stochastic (linear) Lyapunov equations.

References

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