

# Shape and topology optimization of structures built by additive manufacturing

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CMAP, École Polytechnique



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- 1 - Introduction: a review of additive manufacturing
- 2 - Parametric optimization and the adjoint method
- 3 - Geometric optimization and Hadamard method
- 4 - Topology optimization and the level set method
- 5 - Typical constraints from additive manufacturing
- 6 - Optimization of lattice materials
- 7 - Coupled shape and laser path optimization

A "hot" topic with a lot of room for new ideas and modeling...

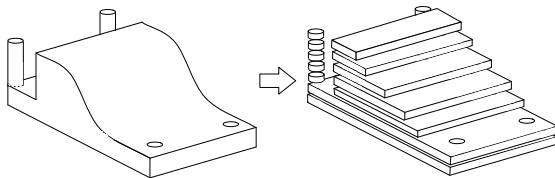
## Chapter 1 - Introduction: a review of additive manufacturing

- I - Principles of additive manufacturing
- II - What can be achieved ?
- III - Some failures of additive manufacturing
- IV - Models of the manufacturing process
- V - Conclusion and perspectives

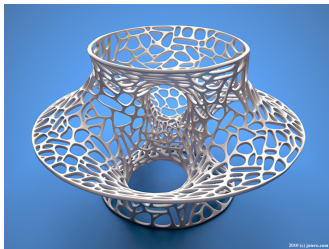


**Sofia project:** Add-Up, Michelin, Safran, ESI, etc. (2016-2022)

- Structures built layer by layer

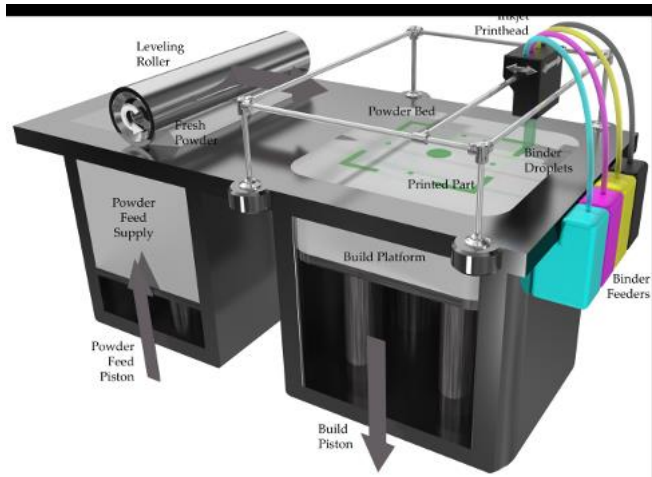


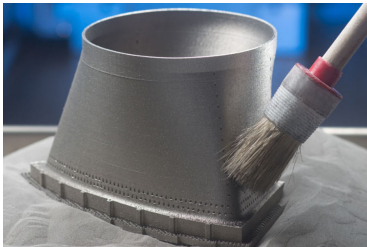
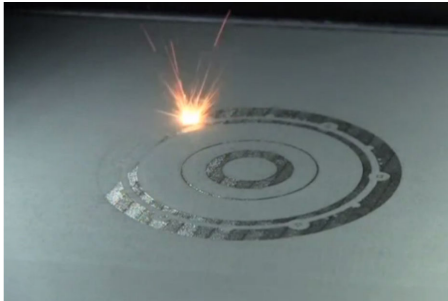
- No topological constraints on the built structures



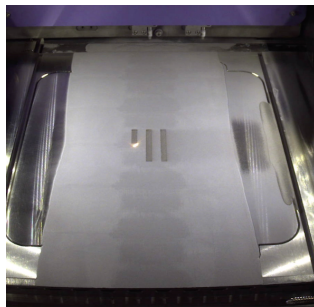
- Various materials: plastic, polymer, metal, ceramic...
- We focus on [metallic additive manufacturing](#).
- Various processes: wire, direct energy deposition (DED), layer by layer...
- We focus on [powder bed techniques](#) (layer by layer).
- Very easy process for building ! A simple [STL file](#) (STereoLithography) is enough for the machine (through a slicing process).

Metallic powder melted by a laser or an electron beam.



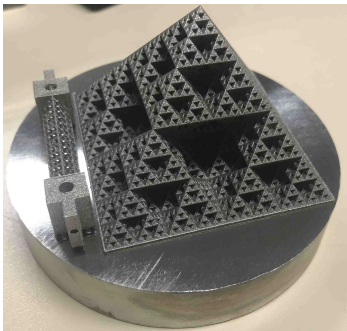


# AddUp machine at LURPA (thanks to C. Tournier)

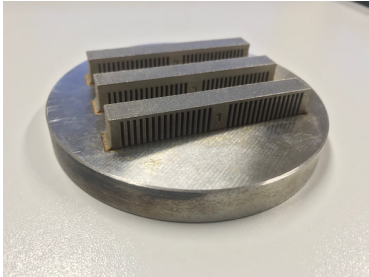


## II - What can be achieved ?

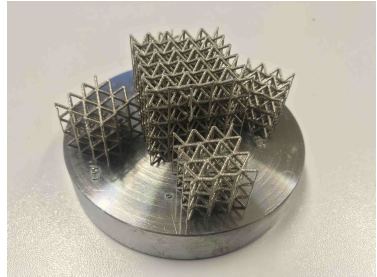
- Very different from classical techniques (molding, milling)
- No topological constraints on the built structures
- Very complicated structures: new applications, new designs
- Lattice (porous) materials
- Functionally graded materials



# Examples from LURPA (thanks to C. Tournier)

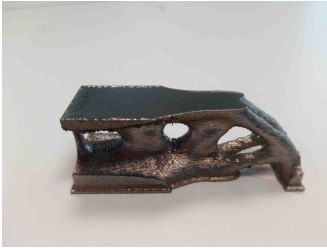


comb-shaped structure



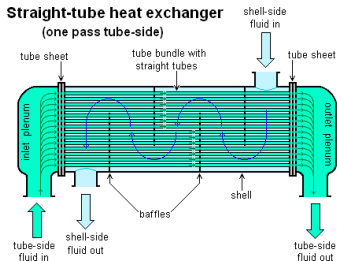
lattice structure

# Examples from SAFRAN (thanks to M. Bihr)

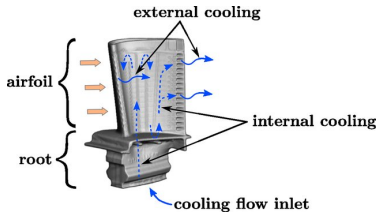


Academic example (MBB beam and its support)

New multi-physics designs can be built. For example:



heat exchanger



turbine blade with internal cooling



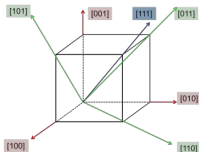
3-d printing enables structures made of composite materials (called **lattice materials**).

Work of Denis Solas, ICMMO, Orsay, Paris-Saclay.

Pilotage de l'anisotropie en fabrication additive par SLM

Texture cristallographique et anisotropie

## Anisotropie élastique d'un monocristal



$$\sigma = C_g : \varepsilon$$

$$C = \begin{pmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44} \end{pmatrix}$$

$$A = \frac{2C_{44}}{C_{11} - C_{12}}$$

	$C_{11}$ (GPa)	$C_{12}$ (GPa)	$C_{44}$ (GPa)	A	$E_{\langle 100 \rangle}$ (GPa)	$E_{\langle 110 \rangle}$ (GPa)	$E_{\langle 111 \rangle}$ (GPa)
Molybdène	457,7	160,9	111,2	0,707	394,4	312,9	292,8
Chrome	350,0	67,8	100,8	0,714	328,0	266,2	250,4
Tungstène	501,0	198,0	151,0	0,997	388,8	388,0	387,7
Aluminium	108,2	61,3	28,5	1,225	63,9	72,6	76,1
Nickel	244,0	158,0	102,0	2,372	119,8	200,6	258,9
Fer alpha	231,4	134,7	116,4	2,407	132,0	220,4	283,3
Cuivre	168,4	121,4	75,4	<b>3,190</b>	<b>66.7</b>	<b>130.3</b>	<b>191,1</b>

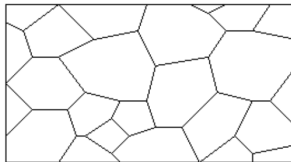
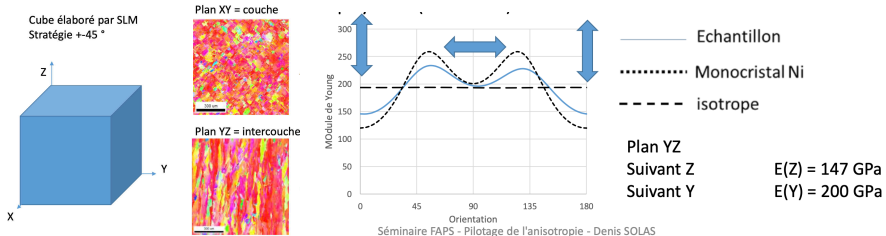
Anisotrope

6/26/2020

Séminaire FAPS - Pilotage de l'anisotropie - Denis SOLAS

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# Functionally graded materials



Polycristal

Work of Denis Solas, ICMMO, Orsay, Paris-Saclay.

## Pilotage de l'anisotropie par SLM



Choix de la stratégie

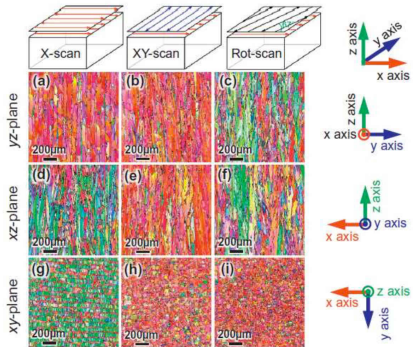
Materials and Design 140 (2018) 307–316

Puissance = 200 W

Vitesse = 800 mm/s

Ecartement = 80  $\mu\text{m}$

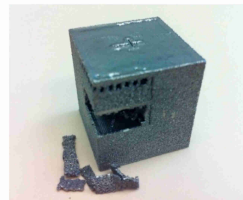
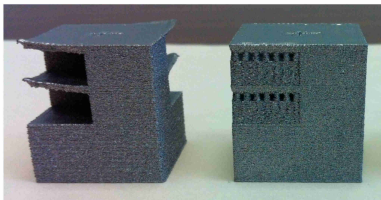
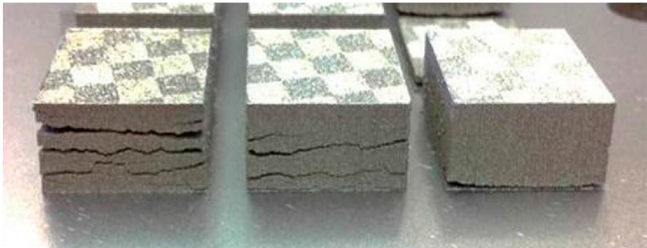
Epaisseur couche = 40  $\mu\text{m}$

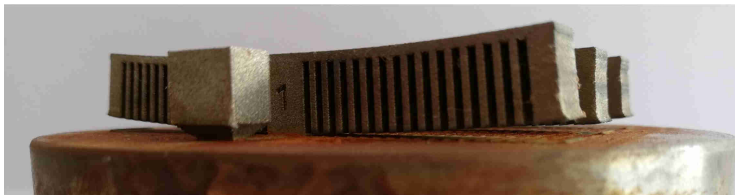
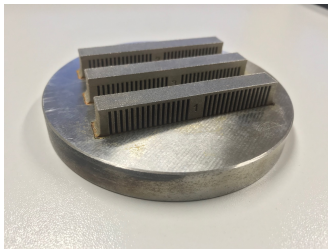


- One can optimize the material properties (anisotropy) by controlling the laser path, its speed and power.
- PhD thesis of Mathilde Boissier (co-supervised with C. Tournier, LURPA): [simultaneous optimization of the path and of the shape](#)
- PhD thesis of Abdelhak Touiti (co-supervised with F. Jouve, LJLL): [simultaneous optimization of the anisotropy and of the shape](#)

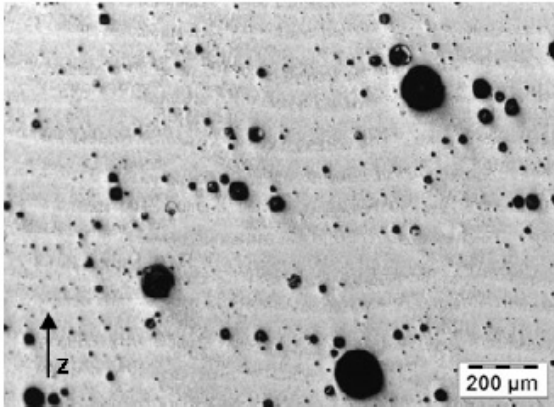
- A very large amount of energy is deposited on the structure: residual thermal stresses.
- Large thermal deformations, thermal fracture.
- Defect, porosities
- Separating a built part from the baseplate is tricky.
- Slow process, not for large series production.

Thermal stresses and deformations:



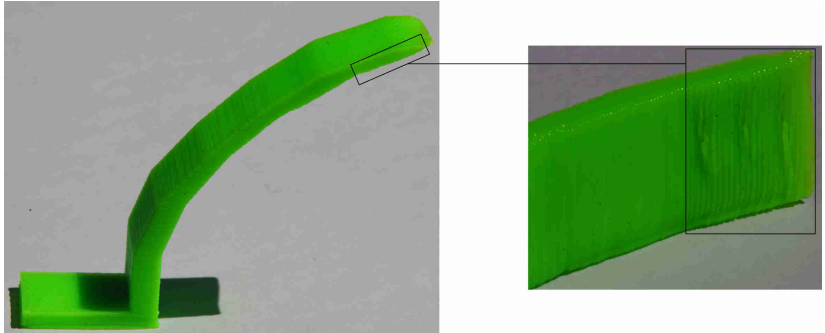


Strong deformation after separation from the baseplate



pores in a 3D-printed aluminum alloy  
(source: Inside Metal Additive Manufacturing)

# Other failure: overhang limitation



The angle between the structural boundary and the build direction has an impact on the quality of the processed shape.



Example of a bad 3-d printing due to overhangs.

Constraints are required to avoid failures in the fabrication process

- almost horizontal **overhang** surfaces cannot be built
- metal melting → large temperatures → **thermal residual stresses** and thermal deformations
- deformations of the structure may stop the powder deposition system
- minimal time (or energy) for completion
- removing the powder (no closed holes)
- bad metallurgical properties (for example, porosities)

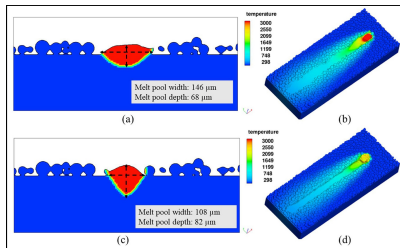
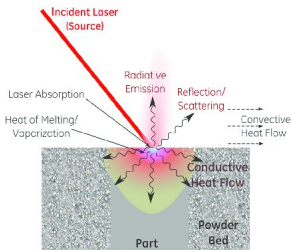
Numerical simulations are required for predicting the success or the failure of the process.

What do we need ?

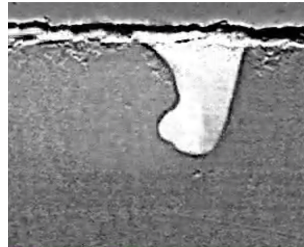
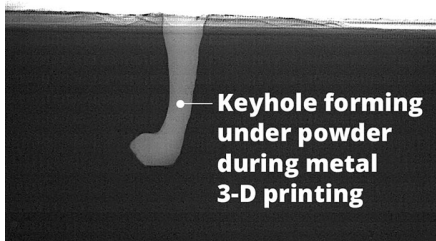
- good models at different length-scales
- multi-physics models
- model reduction and/or HPC
- optimization

and new ideas !

# IV - Models of the manufacturing process



Microscopic model: heat exchange, phase change, fluid mechanics in the melt pool, granular media for the powder coating...  
(Spears & Gold, 2016)



For example: to simulate the "keyhole" phenomenon.

Microscopic models are too computationally intense to be used in optimization loops.

Macroscopic models ignore small details and a lot of physics...  
but they are useful for quick prediction and optimization !

Two examples:

- thermo-mechanical model
- inherent strain model

Heat equation:

$$\left\{ \begin{array}{ll} \rho \frac{\partial T}{\partial t} - \operatorname{div}(\lambda \nabla T) = Q(t) & \text{in } (0, t_F) \times D \\ T = T_{init} & \text{on } (0, t_F) \times \Gamma_{base} \\ \lambda \nabla T \cdot n = -H_e(T - T_{init}) & \text{on } (0, t_F) \times (\partial D \setminus \Gamma_{base}) \\ T(t = 0) = T_{init} & \text{in } D \end{array} \right.$$

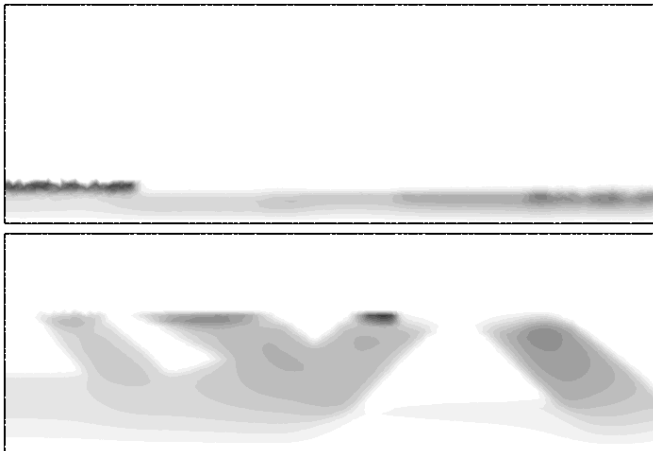
Thermoelastic quasi-static equation:

$$\left\{ \begin{array}{ll} -\operatorname{div}(\sigma) = 0 & \text{and } \sigma = \sigma^{el} + \sigma^{th} \quad \text{in } (0, t_F) \times D, \\ \sigma^{el} = Ae(u) & \text{and } \sigma^{th} = K(T - T_{init}) \operatorname{Id}, \end{array} \right.$$

Material parameters  $\rho, \lambda, A, K$  are different for solid or powder.  
Source term  $Q(t)$  = beam spot, traveling on the upper layer.

**Weak coupling:** **first**, solve the heat equation, **second**, solve thermoelasticity.

# Path of the source term $Q(t)$



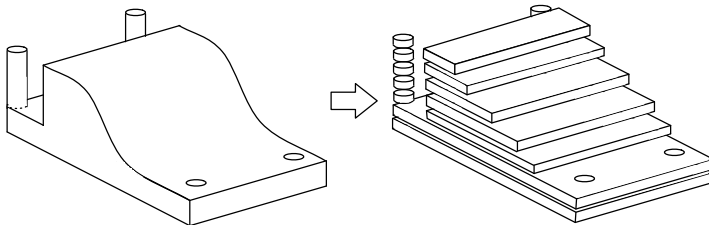
► path

- Simplified engineering model, issued from welding.
- No heat equation !
- Inherent strain  $\varepsilon^{inh}$  tabulated from experiments.

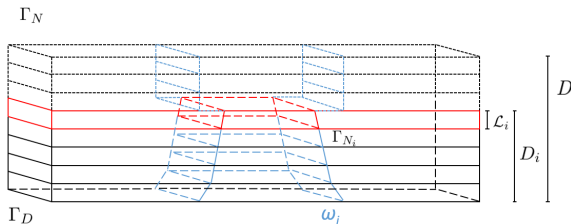
Thermoelastic quasi-static equation:

$$\left\{ \begin{array}{l} -\operatorname{div}(\sigma) = 0 \\ \sigma^{el} = A e(u) \\ + \text{boundary conditions} \end{array} \right. \quad \text{and } \sigma = \sigma^{el} + A \varepsilon^{inh} \quad \text{in } (0, t_F) \times D,$$

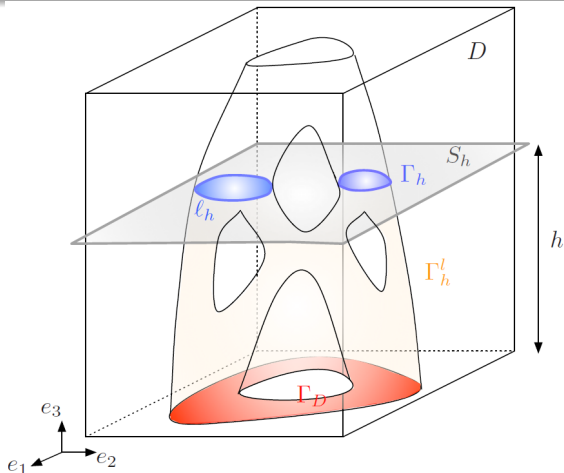
and layer by layer process



Additive manufacturing involves a layer by layer process.  
We must take this process into account.



- Denote by  $D$  the building chamber and by  $\Omega$  the structure to be built.
- In  $\mathbb{R}^d$  the vertical direction,  $e_d$ , is the building direction (layers are normal to  $e_d$ ).
- The layer  $i$  is built at height  $h_i$ .
- When building the  $i$ th layer, only the intermediate domain  $D_i$  and intermediate shape  $\Omega_i$  play a role.



For a final shape  $\Omega$ , define **intermediate shapes**  $\Omega_i$  of increasing height  $h_i$

$$\Omega_i = \{x \in \Omega \text{ such that } x_d \leq h_i\} \quad 1 \leq i \leq n.$$

Rewrite the thermo-elasticity model in a layer by layer context.

- 1 Each layer  $i$  is built between time  $t_{i-1}$  and  $t_i$ ,  $1 \leq i \leq n$ .
- 2 Each layer  $i$  at height  $h_i$ .
- 3 Intermediate domains  $D_i = \{x \in D \text{ such that } x_d \leq h_i\}$

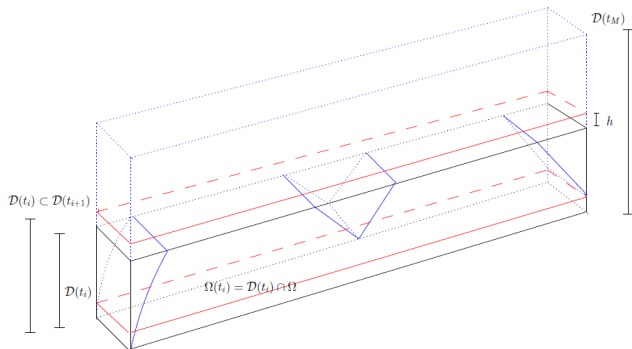
G. Allaire, L. Jakabcsin, *Taking into account thermal residual stresses in topology optimization of structures built by additive manufacturing*, M3AS 28(12), 2313-2366 (2018).

Heat equation:

$$\left\{ \begin{array}{ll} \rho \frac{\partial T}{\partial t} - \operatorname{div}(\lambda \nabla T) = Q(t) & \text{in } (t_{i-1}, t_i) \times D_i \\ T = T_{init} & \text{on } (t_{i-1}, t_i) \times \Gamma_{base} \\ \lambda \nabla T \cdot n = -H_e(T - T_{init}) & \text{on } (t_{i-1}, t_i) \times (\partial D_i \setminus \Gamma_{base}) \\ T(t = t_{i-1}) = T_{init} & \text{in } D_i \setminus D_{i-1} \end{array} \right.$$

Thermoelastic quasi-static equation:

$$\left\{ \begin{array}{ll} -\operatorname{div}(\sigma) = 0 & \text{and } \sigma = \sigma^{el} + \sigma^{th} \quad \text{in } (t_{i-1}, t_i) \times D_i, \\ \sigma^{el} = A e(u) & \text{and } \sigma^{th} = K(T - T_{init}) \operatorname{Id}, \end{array} \right.$$



- 1 Build chamber  $D$ , vertical build direction  $e_d$ .
- 2 Intermediate domains  $D_i = \{x \in D \text{ such that } x_d \leq h_i\}$ .
- 3 Final shape  $\Omega$  and intermediate shapes  $\Omega_i = \Omega \cap D_i$ .
- 4 Mixture  $D_i = \Omega_i \cup P_i$  of solid and powder.

- 1 Powder is neglected and only the intermediate shapes  $\Omega_i = \Omega \cap D_i$  are taken into account.
- 2 The  $i$ th layer is denoted by  $\mathcal{L}_i$ .
- 3 The inherent strain  $\varepsilon^{inh}$  is applied only in the layer  $\mathcal{L}_i$ .

The model is

$$\left\{ \begin{array}{ll} -\operatorname{div}(\sigma_i) &= 0 & \text{in } \Omega_i, \\ \sigma_i &= A(e(u_i) + \varepsilon_{\mathcal{L}_i}) & \text{with } \varepsilon_{\mathcal{L}_i}(x) = \varepsilon^{inh} \chi_{\mathcal{L}_i}(x), \\ \sigma_i n &= 0 & \text{on } \Gamma_{N_i}, \\ u_i &= 0 & \text{on } \Gamma_D \cap \partial\Omega_i. \end{array} \right.$$