Shape and topology optimization of structures built by additive manufacturing

Grégoire ALLAIRE, M. Bihr, B. Bogosel, M. Boissier, C. Dapogny, F. Feppon, A. Ferrer, P. Geoffroy-Donders, M. Godoy, L. Jakabcin, O. Pantz

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Outline of the course

- 1 - Introduction: a review of additive manufacturing
- 2 - Parametric optimization and the adjoint method
- 3 - Geometric optimization and Hadamard method
- 4 - Topology optimization and the level set method
- 5 - Typical constraints from additive manufacturing
- 6 - Optimization of lattice materials
- 7 - Coupled shape and laser path optimization

A "hot" topic with a lot of room for new ideas and modeling...

Chapter 7 - Coupled shape and laser path optimization

SOFIA

Sofia project: Add-Up, Michelin, Safran, ESI, etc. (2016-2022)

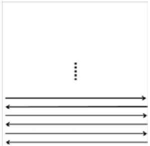
For most (all ?) additive manufacturing machines, the laser path is made of straight lines and on-off processes.

Few attempts to change this strategy and introduce patterns to build better path.

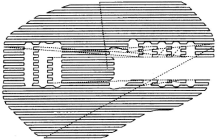
Very few attempts to optimize !

The laser path has a dramatic influence on the quality of the built structure.

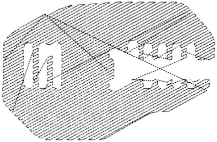
Example of a straight laser path



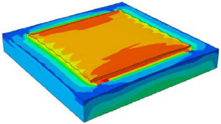
(a) Raster path [52]



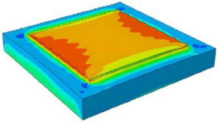
(b) Optimization of the number of jumps thanks to line orientation - direction a [158]



(c) Optimization of the number of jumps thanks to line orientation - direction b [158]

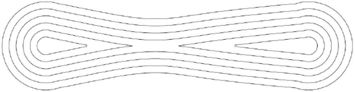


(d) Residual stresses in the X-axis [52]

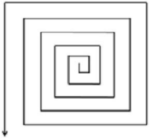


(e) Residual stresses in the Y-axis [52]

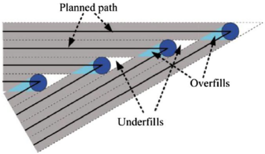
Example of patterns



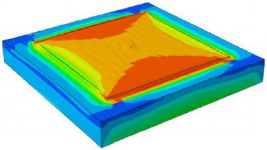
(a) Offset path [108]



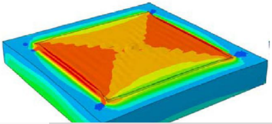
(b) Spiral path [52]



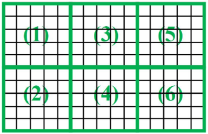
(c) Covering issue [105]



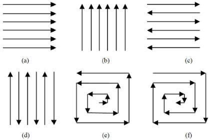
(d) Residual stresses in the X-axis for in-out offsets [52]



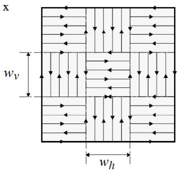
Example of a simple optimization strategy



(a) Splitting of the part's mesh into 6 zones [5]



(b) Strategies to be put in the different zones [5]



(c) Checkerboard strategy [104]

Path optimization based on control theory

Our goal: use control theory to fully optimize the path !

Two different topics:

1. Optimization of the laser path Γ (a curve) in a domain Σ to build a given shape Σ_S.
2. Coupled optimization of the laser path Γ and of the built shape Σ_S.

Very few works ! Tonia-Maria Alam, Serge Nicaise, Luc Paquet, *An optimal control problem governed by the heat equation with nonconvex constraints applied to the selective laser melting process.* Minimax Theory Appl. 6, No. 2, 191-204 (2021).

PhD thesis of Mathilde Boissier (co-advised with C. Tournier, 2020).

References

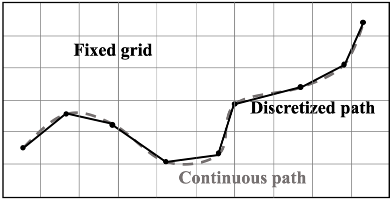
M. Boissier, G. Allaire, C. Tournier, *Additive Manufacturing Scanning Paths Optimization Using Shape Optimization Tools,* SMO, 61, pp. 2437-2466 (2020). HAL preprint: hal-02410481v1 (December 2019).

M. Boissier, G. Allaire, C. Tournier, *Time dependent scanning path optimization for the powder bed fusion additive manufacturing process,* Computer-Aided Design,142, 103122 (2022). HAL preprint: hal-03202102 (April 2021).

M. Boissier, G. Allaire, C. Tournier, *Concurrent shape optimization of the part and scanning path for additive manufacturing,* to appear in SICON. HAL preprint: hal-03124075 (January 2021).

2-d model

- No phase change, no melt pool, no radiation, no non-linearity.
- Averaged model in the built direction: 2-d model.
- Steady state assumption (can be avoided).
- Constant material properties.
- Rectangular domain Σ: built chamber cross-section.
- The path Γ is a smooth connected and open curve.
- Constant power of the laser beam along the path.
- No kinematic constraint.



2-d heat equation

Denote by Γ the laser path. Compute the temperature T solution of

$$\begin{cases} -\operatorname{div}(\lambda \nabla T(x)) + \beta(T(x) - T_{init}) = P\delta_\Gamma(x) & \text{in } \Sigma, \\ \lambda \partial_n T(x) = 0 & \text{on } \partial\Sigma, \end{cases}$$

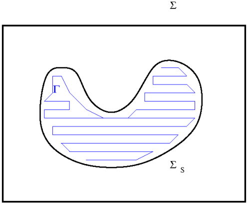
where δ_Γ is the Dirac mass along the path.

- Thermal diffusion $\lambda > 0$ in the plane.
- Thermal loss coefficient $\beta > 0$ out of plane.
- Laser power P.
- Reference temperature T_{init}.

The coefficients are obtained by data assimilation of a 3-d detailed computation.

The goal is to build a given structure $\Sigma_S \subset \Sigma$ with a minimal-length path Γ.

- Criterion for building: $T \geq T_\phi$ in Σ_S where T_ϕ is the fusion temperature.
- Criterion for keeping the powder outside: $T < T_\phi$ in $\Sigma \setminus \Sigma_S$
- Criterion to avoid over-heating: $T < T_{max}$ in Σ_S



Minimize the length of the path with two temperature constraints

$$\min_\Gamma J(\Gamma) = \int_\Gamma ds \qquad \text{such that } C_\phi(T) = C_M(T) = 0$$

Optimization framework

$$\min_{\Gamma} J(\Gamma) = \int_{\Gamma} ds \qquad \text{such that } C_\phi(T) = C_M(T) = 0,$$

where $T = T(\Gamma)$ is the solution of the heat equation. The melting constraint in Σ_S is

$$C_\phi(T) = \int_{\Sigma_S} \left[(T_\phi - T(x))^+ \right]^2 dx$$

and the maximal temperature constraint everywhere in Σ is

$$C_M(T) = \int_{\Sigma} \left[(T(x) - T_M(x))^+ \right]^2 dx$$

with $T_M = T_\phi$ in $\Sigma \setminus \Sigma_S$ and $T_M > T_\phi$ in Σ_S.

Theoretical analysis of this optimization problem

- We did not study existence of optimal path.
- No uniqueness is expected.
- We compute shape derivatives by the adjoint method.

Lemma (classical). Denote by A and B the end points of Γ by n its unit normal vector, by τ its unit tangent vector and by κ its curvature. The shape derivative of $J(\Gamma) = \int_\Gamma ds$ is

$$\langle J'(\Gamma), \theta \rangle = \int_\Gamma \kappa \, \theta \cdot n \, ds + \theta(B) \cdot \tau(B) - \theta(A) \cdot \tau(A)$$

Adjoint

Define the thermal constraint

$$C(\Gamma) = C_\phi(T) + C_M(T) = \int_{\Sigma_S} \left[(T_\phi - T)^+\right]^2 dx + \int_\Sigma \left[(T - T_M)^+\right]^2 dx$$

The corresponding adjoint equation is

$$\begin{cases} -\operatorname{div}(\lambda \nabla T_{\mathrm{adj}}) + \beta T_{\mathrm{adj}} = & 2\Big((T_\phi - T)^+ \chi_{\Sigma_S} \\ & -(T - T_M)^+ \chi_{\Sigma \setminus \Sigma_S} \\ & -(T - T_\phi)^+ \chi_{\Sigma \setminus \Sigma_S} \Big) \quad \text{in } \Sigma, \\ \lambda \partial_n T_{\mathrm{adj}} = 0 & \text{on } \partial\Sigma, \end{cases}$$

where χ_{Σ_S} and $\chi_{\Sigma\setminus\Sigma_S}$ are the characteristic functions of Σ_S and $\Sigma \setminus \Sigma_S$, respectively.

Proposition. Denote by A and B the end points of Γ by n its unit normal vector, by τ its unit tangent vector and by κ its curvature. The shape derivative of the thermal constraint $C(\Gamma)$ is

$$\begin{aligned}\langle C'(\Gamma),\theta\rangle = & -P\int_\Gamma \left(\frac{\partial T_{\text{adj}}}{\partial n} + \kappa T_{\text{adj}}\right)\theta\cdot n\,ds \\ & -PT_{\text{adj}}(B)\theta(B)\cdot\tau(B) + PT_{\text{adj}}(A)\theta(A)\cdot\tau(A)\end{aligned}$$

Proof. Introduce a Lagrangian

$$\begin{aligned}\mathcal{L}(\Gamma,\hat{T},\hat{T_{\text{adj}}}) &= \int_\Sigma \left(\lambda\nabla\hat{T}\cdot\nabla\hat{T_{\text{adj}}} + \beta(\hat{T}-T_{init})\hat{T_{\text{adj}}}\right)dx \\ &\quad -\int_\Gamma P\hat{T_{\text{adj}}}\,ds + C_\phi(\hat{T}) + C_M(\hat{T})\end{aligned}$$

and rely on Céa's method for shape differentiation.

Two ingredients:

1. Regularization of the shape derivative: compute the Riesz representative of $J'(\Gamma)$ or $C'(\Gamma)$ with the $H^1(\Gamma;\mathbb{R}^2)$ scalar product.
2. Augmented Lagrangian algorithm

$$\mathcal{L}(\Gamma,\lambda) = J(\Gamma) + \lambda C(\Gamma) + \frac{1}{2}\mu C(\Gamma)^2$$

with a Lagrange multiplier λ and a penalization parameter $\mu > 0$.

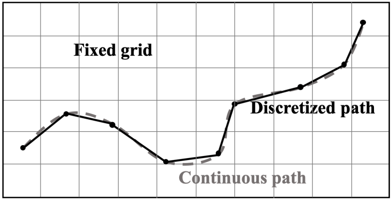
Algorithm:

At each iteration n move Γ by a displacement θ such that $\langle \mathcal{L}'(\Gamma), \theta\rangle \leq 0$. Then, update the Lagrange multiplier

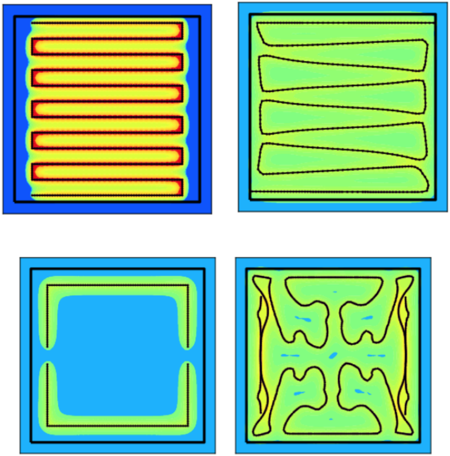
$$\lambda^{n+1} = \lambda^n + \mu C(\Gamma^n)$$

Numerical discretization

1. The mesh of the rectangle Σ is fixed.
2. A discretization of the continuous adjoint is used.
3. The path Γ is discretized by nodes connected by straight lines.
4. At each optimization iteration the nodes are moved by θ.
5. The discretization of Γ can be adapted.
6. Many (local) minima !

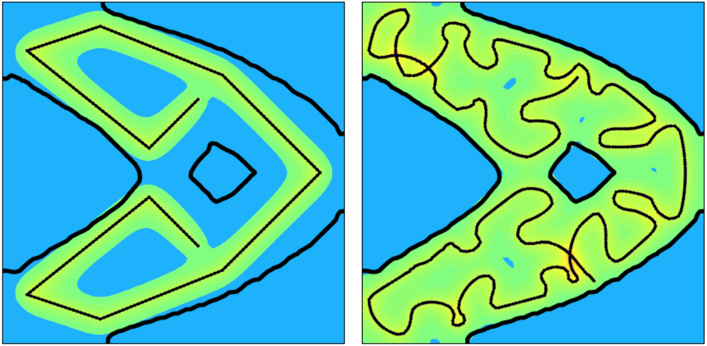


Optimization of laser path only



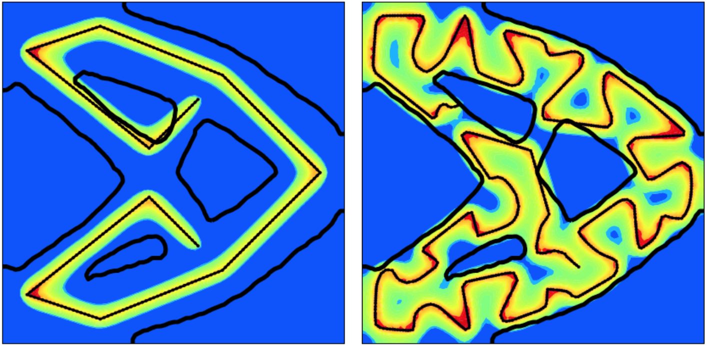
Initialization (left), optimal design (right).
Temperature: blue (cold), red (hot).

Optimization of laser path only



Σ is the square and Σ_S a cantilever
Initialization (left), optimal design (right).

Optimization of laser path only



Σ is the square and Σ_S a cantilever
Initialization (left), optimal design (right).

From now on, denote $\Omega = \Sigma_S$ the shape to build.

To the previous model (heat equation for the path Γ), we add a compliance minimization for the shape Ω.

For a given load $g : \Gamma_N \to \mathbb{R}^2$, the displacement $u : \Omega \to \mathbb{R}^2$ is the solution of

$$\begin{cases} -\operatorname{div}(A\,e(u)) = 0 & \text{in } \Omega \\ u = 0 & \text{on } \Gamma_D \\ \big(A\,e(u)\big)n = g & \text{on } \Gamma_N \\ \big(A\,e(u)\big)n = 0 & \text{on } \Gamma \end{cases}$$

The compliance is

$$C_{\mathrm{ply}}(\Omega) = \int_{\Gamma_N} g \cdot u\, dx$$

The shape Ω appears in the thermal constraints too !

$$\min_{\Omega,\Gamma\subset\Sigma} J(\Omega,\Gamma) = C_{\text{ply}}(\Omega) + \ell \int_\Gamma ds,$$

with the constraints

$$V(\Omega) = V^0, \quad C_\phi(\Omega,\Gamma) = 0, \quad C_M(\Omega,\Gamma) = 0$$

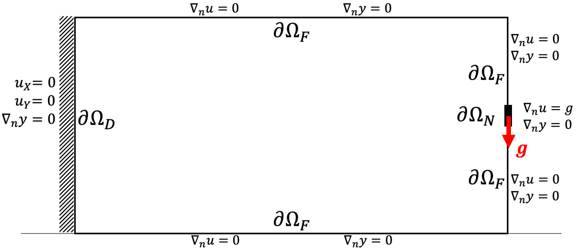
where, T_ϕ being the melting temperature,

$$C_\phi(\Omega,\Gamma) = \int_\Omega \left[(T_\phi - T(x))^+\right]^2 dx$$

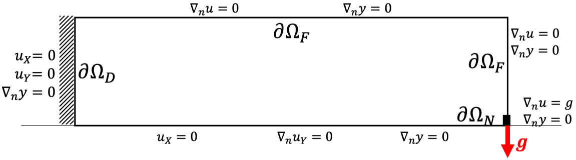
and the maximal temperature T_M is different inside and outside Ω,

$$C_M(\Omega,\Gamma) = \int_\Sigma \left[(T(x) - T_M(\Omega,x))^+\right]^2 dx$$

where T is the solution of the heat equation in Σ.

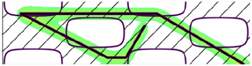


Cantilever test case



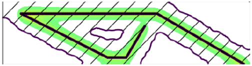
Half domain using an horizontal symmetry

Same case for different strategies

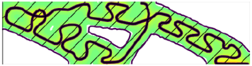


(a) Initialization (INI)

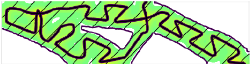
(b) Shape only (SONLY)



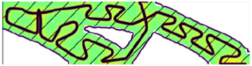
(c) Shape path (STEMP)



(d) Coupled from initialization (SP-fromINI)

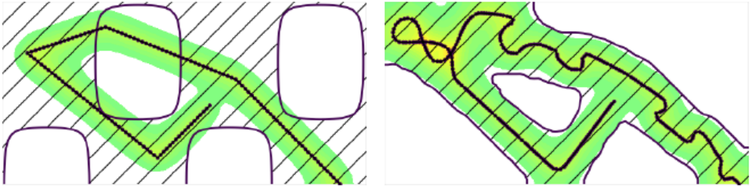


(e) Path only (PONLY)



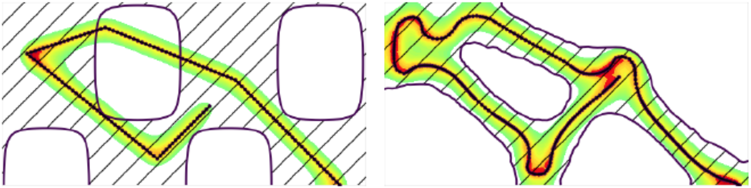
(f) Coupled from path only (SP-fromPONLY)

Same case for two different materials



Initialization

Result



Initialization

Result

Same case for a larger cantilever

Initialization (left), optimal shape for compliance only (right).

Coupled optimal design from initialization (left), optimal path for the fixed optimal shape (right).

Compute the temperature T solution of

$$\begin{cases} \rho c_p \dfrac{\partial T}{\partial t} - \operatorname{div}(\lambda \nabla T) + \beta(T - T_{init}) = q(t,x) & \text{in } \Sigma, \\ \lambda \partial_n T = 0 & \text{on } \partial\Sigma, \end{cases}$$

with a moving source term with spot radius r

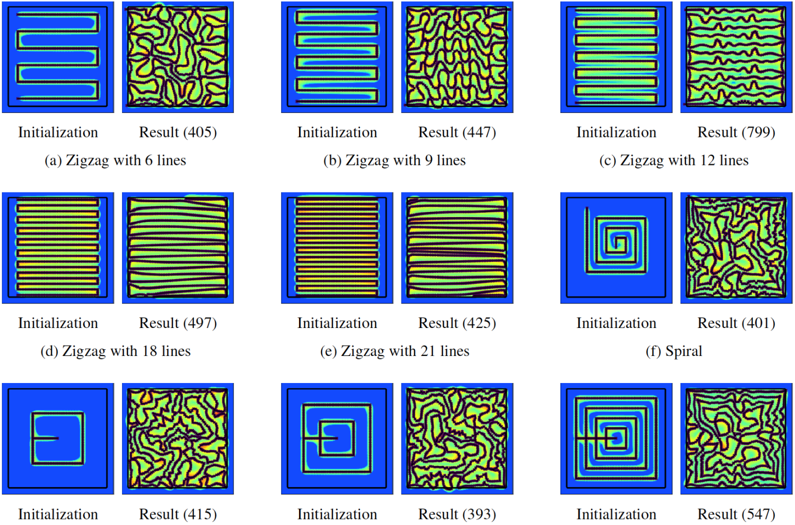
$$q(t,x) = P \exp\left(-\frac{|x - u(t)|^2}{r^2}\right) \quad 0 \leq t \leq t_{final}$$

and the focus point is a solution of the ODE

$$\begin{cases} \dot{u}(t) = V\tau(t) & t \in (0, t_{final}) \\ u(0) = x_0. \end{cases}$$

Similar optimization problem: much more costly (backward adjoint).

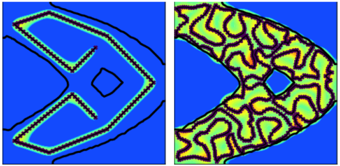
Unsteady numerical results



(a) Zigzag with 6 lines (b) Zigzag with 9 lines (c) Zigzag with 12 lines

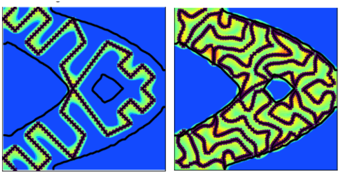
(d) Zigzag with 18 lines (e) Zigzag with 21 lines (f) Spiral

(g) Contour with 1 line (h) Contour with 2 lines (i) Contour with 4 lines



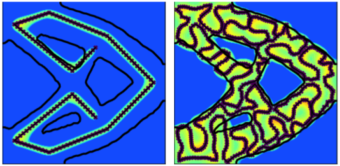
Initialization Result (iteration 419)

(a) Initialization 1



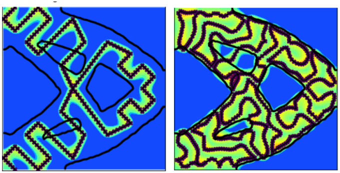
Initialization Result (iteration 466)

(b) Initialization 2



Initialization Result (iteration 417)

(a) Initialization 1



Initialization Result (iteration 459)

(b) Initialization 2

Further generalizations

- Variable power P along the path.
- Penalization of P towards its maximal value or 0, with a total variation constraint: topology optimization of the path.

See the PhD thesis of Mathilde Boissier
Coupling structural optimization and trajectory optimization methods in additive manufacturing, Institut Polytechnique de Paris, december 2020.